Derivation of Bohr Orbit Energy E_n in terms of a_o

Derivation of Bohr orbit energy, E_n in terms of a_o (first Bohr radius).

$$a_o = \frac{h^2 \epsilon_o}{\pi e^2 m}$$

$$E_n = -\frac{e^4 m}{8\epsilon_o^2 h^2 n^2} = -\frac{e^2}{8\pi\epsilon_o n^2}\left(\frac{\pi e^2 m}{h^2 \epsilon_o}\right)$$

where $\left(\frac{\pi e^2 m}{h^2 \epsilon_o}\right) = \frac{1}{a_o}$

$$\therefore E_n = -\frac{e^2}{8\pi\epsilon_o n^2 a_o}$$

$n = 1$ $\quad \therefore E_1 = -\frac{(1.602 \times 10^{-19})^2}{8\pi(8.84 \times 10^{-12})(1)^2(0.529 \times 10^{-10} m)}$

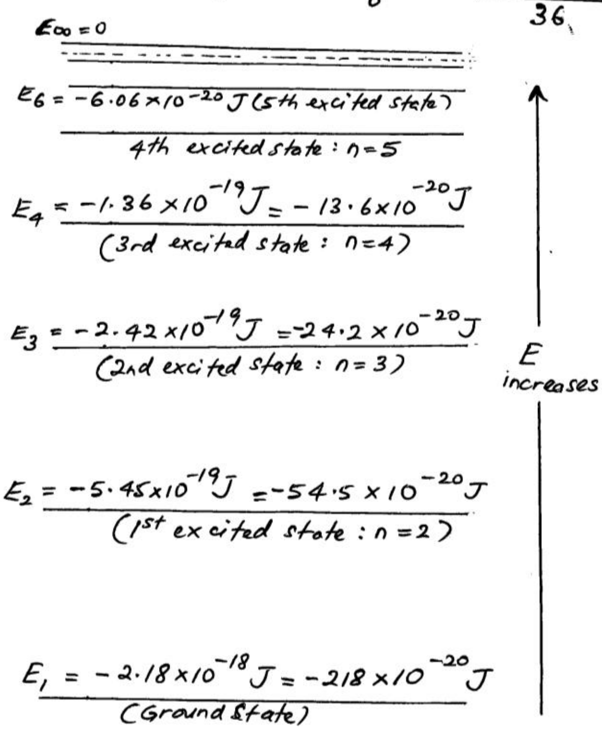
$E_1 = -2.18 \times 10^{-18}$ J

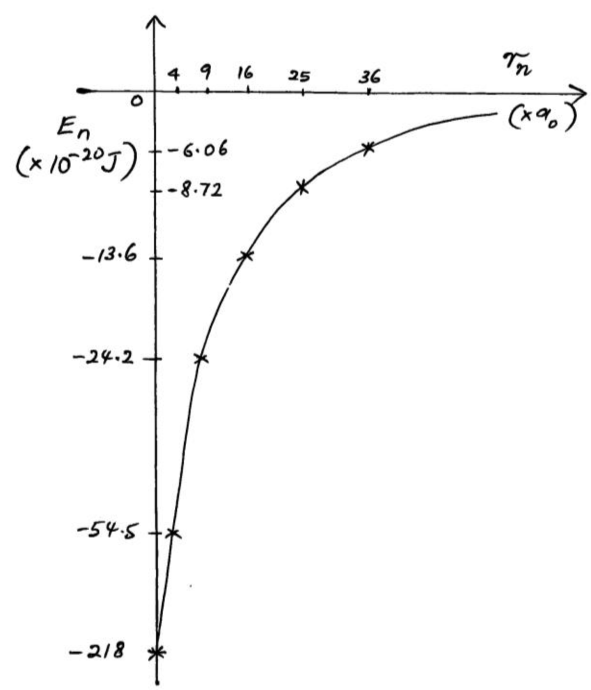
$n = 2$ $\quad E_2 = -\frac{2.18 \times 10^{-18}}{2^2} = -\frac{2.18 \times 10^{-18}}{4} = -5.45 \times 10^{-19}$ J

$n = 3$ $\quad E_3 = \frac{-2.18 \times 10^{-18}}{9} = -2.42 \times 10^{-19}$ J

$n = 4$ $\quad E_4 = -\frac{2.18 \times 10^{-18}}{16} = -1.36 \times 10^{-19}$ J

$n = 6$ $\quad E_6 = -\frac{2.18 \times 10^{-18}}{36} = -6.06 \times 10^{-20}$ J





Derivation of Energy Difference, △E in terms of a_o

$$E_n = -\frac{e^2}{8\pi\epsilon_o a_o n^2}$$

$$\Delta E = E_y - E_x \qquad E_y > E_x$$

$$\Delta E = \frac{e^2}{8\pi\epsilon_o a_o}\left[\frac{1}{n_x^2} - \frac{1}{n_y^2}\right]$$

Problem-solving Example 13

Calculate the energy difference, ΔE between the first energy level (ground state) and the seventh energy level in hydrogen atom using the data given.

[$a_o = 0.529$ Å ; $e = 1.602 \times 10^{-19}$ C ; $\epsilon_o = 8.84 \times 10^{-12} N^{-1} m^{-2} C^2$]

Solution:

$$\Delta E = \frac{e^2}{8\pi\epsilon_o a_o}\left(\frac{1}{n_x^2} - \frac{1}{n_y^2}\right)$$

$a_o = 0.529$ Å
$= 0.529 \times 10^{-10}$ m
$n_x = 1$
$n_y = 7$

$$\Delta E = \frac{(1.602 \times 10^{-19})^2}{8\pi(8.84 \times 10^{-12})(0.529 \times 10^{-10} m)}\left(\frac{1}{1^2} - \frac{1}{7^2}\right)$$

$$\Delta E = \frac{2.57 \times 10^{-38}}{1.18 \times 10^{-20}}\left(1 - \frac{1}{49}\right)$$

$$= 2.178 \times 10^{-18}(0.979)$$

$$\Delta E = 2.134 \times 10^{-18} \text{ J photon}^{-1} \text{ (Ans)}.$$

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