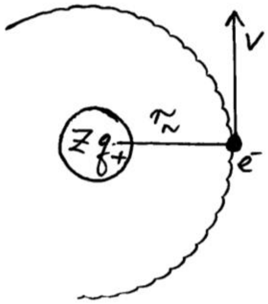
Derivation of Bohr orbit Energy, E_n and radius, r_n for HYDROGEN-LIKE ATOM

DERIVATION OF E_n and r_n FOR HYDROGEN-LIKE ATOM (SINGLE ELECTRON SYSTEM)

$$\left.\begin{array}{l} He^{+}(1e^-, 2p^+)\ ;\ Li^{2+}(1e^-, 3p^+)\ ;\ Be^{3+}(1e^-, 4p^+) \\ B^{4+}(1e^-, 5p^+)\ ;\ C^{5+}(1e^-, 6p^+) \end{array}\right\} \text{different } Z \text{ values.}$$



$$F = \frac{(-e)(+Ze)}{4\pi\epsilon_0 r_n^2} \quad \leftarrow \quad F = \frac{1}{4\pi\epsilon_0}$$

$$F = -\frac{Ze^2}{4\pi\epsilon_0 r_n^2} \quad \text{-----(i)}$$

$$F' = -\frac{mv^2}{r_n} \quad \text{------(ii)}$$

(centrifugal force)

(i) = (ii) :

$$F = F'$$

$$-\frac{Ze^2}{4\pi\epsilon_0 r_n^2} = -\frac{mv^2}{r_n}$$

$$\therefore\ mv^2 = \frac{Ze^2}{4\pi\epsilon_0 r_n} \quad \text{------(iii)}$$

Total Energy, E_n = Potential Energy + Kinetic Energy

$$= (F \times r_n) + \left(\tfrac{1}{2}mv^2\right)$$

$$\text{Total Energy, } E_n = \text{Potential Energy} + \text{Kinetic Energy}$$

$$= (F \times r_n) + \left(\frac{1}{2} m v^2\right)$$

$$= -\frac{e^2 Z}{4\pi \epsilon_0 r_n} + \frac{1}{2}\left(\frac{e^2 Z}{4\pi\epsilon_0 r_n}\right)$$

$$E_n = -\frac{1}{2}\frac{e^2 Z}{4\pi\epsilon_0 r_n} \quad \text{......} (iv)$$

angular momentum

$$m v r_n = n\frac{h}{2\pi} \Rightarrow m^2 v^2 r_n^2 = \frac{n^2 h^2}{4\pi^2}$$

$$(mv^2) m r_n^2 = \frac{n^2 h^2}{4\pi^2}$$

$$\frac{Z e^2}{4\pi\epsilon_0 r_n} \times m r_n^2 = \frac{n^2 h^2}{4\pi^2}$$

$$\therefore r_n = \frac{n^2 h^2 \epsilon_0}{\pi e^2 m Z} \quad \text{....} (v)$$

From eq$\underline{n}$ (iv) & (v) substitute for r_n.

$$E_n = -\frac{e^4 m Z^2}{8 \epsilon_0^2 h^2 n^2} \quad \text{.....} (vi)$$

Prepared by
V.Manoharan
vmano@usm.my
manovv1955@yahoo.com
04-6533888 ext 3566