Derivation of Bohr Orbit Radius r_n

Electron motion is limited at specific orbit around the nucleus. The allowed orbits are orbits where the electron's angular momentum, mvr_n is the multiple integer of $h/2\pi$, ie.

$$mvr_n = n\frac{h}{2\pi} \quad \text{.......(v)}$$

where $n = 1, 2, 3 \ldots$ (quantum number)

* As long as a electron at a allowed orbit, it doesnot radiate or absorb any energy. These type of orbits are called as <u>stationary orbits</u> or <u>stationary states</u>

$$mv^2 = \frac{e^2}{4\pi\epsilon_o r_n} \quad \text{.......(iii)}$$

$$mvr_n = \frac{nh}{2\pi} \quad \text{......(v)}$$

$$[\text{Eqn (v)}]^2 \rightarrow \underbrace{(mv^2)}\, m r_n^2 = \frac{n^2h^2}{4\pi^2}$$

$$\left(\frac{e^2}{4\pi\epsilon_o r_n}\right) m r_n^2 = \frac{n^2h^2}{4\pi^2}$$

$$\therefore r_n = \frac{n^2h^2\epsilon_o}{\pi e^2 m} \quad \text{......(vi)}$$

in SI unit.

$\epsilon_o = 8.84 \times 10^{-12}\ N^{-1}m^{-2}C^2$

$m = m_e = 9.1098 \times 10^{-31}\ kg$

$e = -1.60217 \times 10^{-19}\ C$

$h = 6.626 \times 10^{-34}\ Js$

$n = 1, 2, 3, \ldots$ (quantum no.)

r_n = radius of the n^{th} orbit.

From equation (iv) and (vi):

$$E_n = -\frac{1}{2}\frac{e^2}{4\pi\epsilon_0 r_n} \quad \text{and} \quad r_n = \frac{n^2 h^2 \epsilon_0}{\pi e^2 m}$$

Substitute for r_n:

$$\text{or} \quad \frac{1}{r_n} = \frac{\pi e^2 m}{n^2 h^2 \epsilon_0}$$

$$\therefore E_n = -\frac{e^2}{8\pi\epsilon_0}\left(\frac{1}{r_n}\right) = -\frac{e^2}{8\pi\epsilon_0}\left(\frac{\pi e^2 m}{n^2 h^2 \epsilon_0}\right)$$

$$E_n = -\frac{e^4 m}{8\epsilon_0^2 h^2 n^2}$$

$$\Delta E = \frac{e^4 m}{8\epsilon_0^2 h^2}\left[\frac{1}{n_x^2} - \frac{1}{n_y^2}\right]$$

substitute for e, m, ϵ_0 and h:

$$\Delta E = 2.18 \times 10^{-18}\left[\frac{1}{n_x^2} - \frac{1}{n_y^2}\right]$$

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