
UNIVERSITI SAINS MALAYSIA

First Semester Examination
Academic Session 2011/2012

January 2012

KAA 501 – Quality Control In Chemistry
[Kawalan Mutu Dalam Kimia]

Duration : 3 hours
[Masa : 3 jam]

Please check that this examination paper consists of TWENTY FIVE pages of printed material before you begin the examination.

Instructions:

Answer **FIVE** (5) questions. **Section A** is **COMPULSORY**. Answer **TWO** (2) questions from **Section B**. If a candidate answers more than five questions only the first five questions in the answer sheet will be graded.

Answer each question on a new page.

You may answer the questions either in Bahasa Malaysia or in English.

In the event of any discrepancies, the English version shall be used.

Appendices: AQL Sampling Table based on the Mil-STD-105D

Cumulative standard normal distribution (values of the probability ϕ corresponding to the value Z_ϕ of a standard normal random variable)

Plackett and Burman Design for 11 factors

Constants for the construction of control charts

Table of the Standard Normal Cumulative distribution Function $\Phi(z)$

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Section A**COMPULSORY** questions.

1. A comparison between three methods using three different test kits for the determination of chemical oxygen demand (COD) in water samples was performed. The data (in mg L⁻¹) are given below:

| Method A | Method B | Method C |
|--------------------|-------------------|-------------------|
| 262 | 257 | 266 |
| 260 | 257 | 269 |
| 255 | 259 | 266 |
| 257 | 254 | 264 |
| 259 | 254 | 267 |
| 269 | 258 | 279 |
| 264 | 257 | 295 |
| 271 | 258 | 279 |
| 265 | 261 | 266 |
| 262 | 257 | 266 |
| $\bar{X} = 262.4$ | $\bar{X} = 257.2$ | $\bar{X} = 271.7$ |
| SS= 688766.00 | SS= 661558.00 | SS= 739077.00 |
| Grand mean = 263.8 | | |

- (i) Prove that there is a significant difference among the methods at significance level of 5%.
- (ii) Use Scheffe's test to identify among the methods that give the significant difference as observed in part (i).
- (iii) If Method A is considered to be the standard method, find out which of the other two methods (B or C) that is the closest to the standard method using Dunnet's test.
- (20 marks)
2. (a) Explain the causes of the systematic and random effects in sampling. Discuss the measures to reduce these effects in sampling.
- (8 marks)

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- (b) In the production of infant porridge, vitamin A together with vitamin C and D are added as a premix which is a minor ingredient. All ingredients are mixed thoroughly before distribution into packages. In the determination of vitamin A content, a measurement uncertainty of 10% would be considered acceptable. Two samples (S1, S2) were collected from each of 8 production units (batches) of one type of baby porridge powder. The samples were collected in 50 g size and each sample was analysed for vitamin A in duplicate. The results of vitamin A content ($\mu\text{g g}^{-1}$) for the 50-g samples are shown in Table 1.

Table 1

| Batch | S1A1 | S1A2 | S2A1 | S2A2 |
|--------------|-------------|-------------|-------------|-------------|
| B1 | 403 | 326 | 362 | 352 |
| B2 | 383 | 320 | 350 | 363 |
| B3 | 332 | 292 | 398 | 349 |
| B4 | 281 | 279 | 359 | 322 |
| B5 | 371 | 410 | 379 | 461 |
| B6 | 345 | 320 | 382 | 393 |
| B7 | 298 | 334 | 342 | 316 |
| B8 | 337 | 321 | 293 | 307 |

Calculate the measurement, analytical and sampling uncertainties. Comment on the results.

(12 marks)

3. (a) The data shown below are \bar{X} values of size $n = 5$ taken from an analytical process for the daily monitoring of lead (Pb) content ($\mu\text{g L}^{-1}$) in water samples from a river.

| Day | \bar{X} | Day | \bar{X} |
|-----|-----------|-----|-----------|
| 1 | 7.2 | 13 | 8.2 |
| 2 | 6.2 | 14 | 5.5 |
| 3 | 3.4 | 15 | 2.8 |
| 4 | 6.8 | 16 | 3.5 |
| 5 | 4.8 | 17 | 4.5 |
| 6 | 8.2 | 18 | 4.8 |
| 7 | 4.0 | 19 | 4.2 |
| 8 | 3.6 | 20 | 7.8 |
| 9 | 4.4 | 21 | 7.4 |
| 10 | 3.8 | 22 | 4.4 |
| 11 | 5.6 | 23 | 4.2 |
| 12 | 3.6 | 24 | 3.6 |

- (i) Set up \bar{X} chart of this analytical process.
- (ii) Is the process under statistical control? Provide an explanation for your answer. (10 marks)
- (b) The design specification of a pressed drug tablet calls for a target weight of 2500 mg, with a tolerance of ± 44.2 mg. The process is stable with a mean of 2498 mg and the $USL = 2650$ mg and $LSL = 2350$ mg.
- (i) Define the meanings of C_p and C_{pk} . What is the value of C_p and C_{pk} for this design?
- (ii) Assuming that the output is normally distributed, what proportion of tablets can be expected to be non-conforming? (10 marks)

Section B

Answer **TWO** (2) questions

4. (a) You are a consultant to an analytical laboratory that wants to do the analysis of illicit drugs in body fluids provided by a legal client. However, many of their equipment have regular breakdowns.
- (i) Describe the use of Ishikawa diagram of the magnificent seven tools in statistical process control for you to find the root cause of this problem.
 - (ii) The manager of this laboratory decides to replace the high performance liquid chromatography (HPLC) instrument. Discuss various factors that he should consider in trying to aid his decision making.
- (10 marks)
- (b) Acceptance sampling involves the application of a predetermined plan to decide whether a batch of goods meets the defined criteria for acceptance.
- (i) Describe the use of operating characteristics curve in acceptance sampling.
 - (ii) Compare single and double sampling plans.
- (10 marks)
5. (a) The data for the daily monitoring of lead (Pb) content of water samples from a river are given in the Table below. The data have also been processed for drawing rudimentary CUSUM chart.
- (i) Explain the concept of a CUSUM chart and provide its advantages over the \bar{X} chart.
 - (ii) Fill in the blanks in the table.
 - (iii) Provide a rudimentary CUSUM plot for this analytical process based on the provided Table.

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| Day | \bar{X} | Z_i | C_i |
|-----|-----------|-------|-------|
| 1 | 7.2 | 1.83 | 1.83 |
| 2 | 6.2 | | |
| 3 | 3.4 | -1.97 | 0.69 |
| 4 | 6.8 | 1.43 | 2.12 |
| 5 | 4.8 | -0.57 | 1.55 |
| 6 | 8.2 | | |
| 7 | 4.0 | -1.37 | 3.01 |
| 8 | 3.6 | -1.77 | 1.24 |
| 9 | 4.4 | -0.97 | 0.27 |
| 10 | 3.8 | | |
| 11 | 5.6 | 0.23 | -1.07 |
| 12 | 3.6 | -1.77 | -2.84 |
| 13 | 8.2 | 2.83 | -0.01 |

(b) Differentiate between the following method validation terms:

- (i) Specificity and selectivity
- (ii) Reproducibility and repeatability
- (iii) Intermediate precision and instrument precision
- (iv) Sensitivity and linearity

(20 marks)

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6. Faber et al. (Journal of Chromatography A, 897 (2000) 329–338) reported the use of two-level Plackett-Burman designs to carry out the ruggedness test for a capillary electrophoresis method using the “short-end injection” technique used for the dissolution kinetics of calcium acamprostate from enteric-coated tablets. Eight potentially critical factors were selected including three dummy factors. The calculated effects using N = 12 design are given in the table below.

| Factors | Effects | Rankit |
|----------------|---------|--------|
| Acamprostate | 0.041 | 0.06 |
| Voltage | -0.113 | 0.17 |
| Sorbate | -0.072 | 0.29 |
| Injection time | -0.0613 | 0.41 |
| Wavelength | 0.015 | 0.53 |
| Rinse time | 0.046 | 0.67 |
| Temperature | -0.121 | 0.81 |
| Borate | 0.461 | 0.98 |
| Dummy 1 | 0.0072 | 1.19 |
| Dummy 2 | 0.034 | 1.45 |
| Dummy 3 | 0.022 | 1.91 |

- (i) Use the half normal plot to identify the significant effects if any and the corresponding factors of the ruggedness test above.
- (ii) Use the algorithm of Dong to estimate the error and calculate the margin of error (ME) at $\alpha = 0.05$.
- (iii) Calculate E_{critical} based on the error estimated from the dummy factors and identify the significant effects if any and their corresponding factors. Are there any anomalies between the results of (ii) and (iii)?

(20 marks)

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7. (a) According to USEPA guidelines, all laboratories that use graphite furnace to determine lead in drinking water must achieve a detection limit of $1.5 \mu\text{g L}^{-1}$. A sample solution containing dilute nitric acid and $5 \mu\text{g L}^{-1}$ of lead was prepared. The calibration curve was prepared using 4 standards at 5, 10, 20, $50 \mu\text{g L}^{-1}$ and a blank. Seven replicates of the sample were analysed according to a specified method. The results (in $\mu\text{g L}^{-1}$) are given in the following table.

| Sample number | Results ($\mu\text{g L}^{-1}$) | Recovery (%) |
|---------------|----------------------------------|--------------|
| blank | -0.8 | NA |
| 1 | 4.9 | 98 |
| 2 | 4.7 | 94 |
| 3 | 4.6 | 92 |
| 4 | 4.5 | 90 |
| 5 | 4.7 | 94 |
| 6 | 4.8 | 96 |
| 7 | 4.8 | 96 |

NA: Not applicable

- (i) Differentiate the terms method detection limit (MDL) and limit of quantification n (LOQ).
- (ii) Determine the method detection limit (MDL) and limit of quantitation (LOQ) for this method. Provide appropriate verification of the obtained MDL value.

(10 marks)

- (b) The ISO quality standards have been established to provide quality management of the laboratory and industries as well as generating reliable quality products or services.

- (i) What are the key differences between accreditation and certification?
- (ii) Outline the normal procedure and preparation for your laboratory to obtain ISO 17025 accreditation.

(10 marks)

Bahagian ASoalan **WAJIB**

1. Satu perbandingan tiga kaedah yang menggunakan tiga kit ujian yang berbeza untuk penentuan tuntutan kimia oksigen (COD) sampel air telah dilakukan. Data (mg L^{-1}) diberikan seperti berikut:

| Kaedah A | Kaedah B | Kaedah C |
|----------------------|-------------------|-------------------|
| 262 | 257 | 266 |
| 260 | 257 | 269 |
| 255 | 259 | 266 |
| 257 | 254 | 264 |
| 259 | 254 | 267 |
| 269 | 258 | 279 |
| 264 | 257 | 295 |
| 271 | 258 | 279 |
| 265 | 261 | 266 |
| 262 | 257 | 266 |
| $\bar{X} = 262.4$ | $\bar{X} = 257.2$ | $\bar{X} = 271.7$ |
| SS= 688766.00 | SS= 661558.00 | SS= 739077.00 |
| Purata Utama = 263.8 | | |

- (i) Buktikan terdapat perbezaan ketara di antara kaedah tersebut pada paras signifikan 5%.
- (ii) Guna ujian Scheffe untuk mengenalpasti kaedah manakah yang mempunyai perbezaan ketara seperti yang diperolehi pada ujian dalam bahagian (i).
- (iii) Sekiranya Kaedah A dianggap sebagai kaedah piawai, tentukan yang manakah di antara kaedah B dan C yang paling menyamai kaedah piawai tersebut melalui ujian Dunnett.
- (20 markah)
2. (a) Terangkan sebab bagi kesan sistematik dan rambang dalam pensampelan. Bincangkan langkah untuk mengurangkannya.

(8 markah)

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- (b) Dalam penghasilan bubur bayi, vitamin A bersama vitamin C dan D ditambah sebagai pra-campuran yang merupakan bahan minor. Semua bahan dicampur dengan lengkap sebelum ditaburkan kepada bungkus. Dalam penentuan kandungan vitamin A, ketidakpastian penyukatan sebanyak 10% dianggap dapat diterima. Dua sampel (S1, S2) telah dikutip daripada setiap 8 unit penghasilan (kelompok) bagi sejenis serbuk bubur bayi. Sampel itu dikutip dalam saiz 50 g dan setiap sampel dianalisis dua kali. Keputusan kandungan vitamin A ($\mu\text{g g}^{-1}$) bagi sampel 50 g ditunjukkan dalam Jadual 1.

Jadual 1

| Kelompok | S1A1 | S1A2 | S2A1 | S2A2 |
|-----------------|-------------|-------------|-------------|-------------|
| B1 | 403 | 326 | 362 | 352 |
| B2 | 383 | 320 | 350 | 363 |
| B3 | 332 | 292 | 398 | 349 |
| B4 | 281 | 279 | 359 | 322 |
| B5 | 371 | 410 | 379 | 461 |
| B6 | 345 | 320 | 382 | 393 |
| B7 | 298 | 334 | 342 | 316 |
| B8 | 337 | 321 | 293 | 307 |

Kirakan ketidakpastian penyukatan, analisis dan pensampelan. Berikan ulasan terhadap keputusan itu.

(12 markah)

3. (a) Data yang diberikan di bawah adalah nilai \bar{X} -bar bagi saiz $n=5$ yang diambil daripada suatu proses pemmonitoran kandungan plumbum (Pb) dalam sampel air sebuah sungai.

| Hari | \bar{X} | Hari | \bar{X} |
|------|-----------|------|-----------|
| 1 | 7.2 | 13 | 8.2 |
| 2 | 6.2 | 14 | 5.5 |
| 3 | 3.4 | 15 | 2.8 |
| 4 | 6.8 | 16 | 3.5 |
| 5 | 4.8 | 17 | 4.5 |
| 6 | 8.2 | 18 | 4.8 |
| 7 | 4.0 | 19 | 4.2 |
| 8 | 3.6 | 20 | 7.8 |
| 9 | 4.4 | 21 | 7.4 |
| 10 | 3.8 | 22 | 4.4 |
| 11 | 5.6 | 23 | 4.2 |
| 12 | 3.6 | 24 | 3.6 |

Sediakan carta x-bar untuk proses analisis ini. Adakah proses ini di dalam keadaan terkawal statistik? Sekiranya tidak, terangkan kenapa.

(10 markah)

- (b) Spesifikasi suatu rekabentuk tablet ubat tertekan memerlukan berat sasaran 2500 mg, dengan nilai toleransi ± 44.2 mg. Proses adalah stabil dengan nilai purata 2498 mg dan $USL = 2650$ mg dan $LSL=2350$ mg.

- (i) Definisikan erti C_p dan C_{pk} . Apakah nilai C_p dan C_{pk} untuk rekabentuk ini.
- (ii) Sekiranya taburan pengeluaran bersifat normal, apakah nisbah tablet yang dijangka tidak memenuhi spesifikasi ?

(10 markah)

Bahagian B

Jawab **DUA** (2) soalan

4. (a) Anda adalah seorang pakar runding kepada sebuah makmal analisis yang mahu melakukan analisis terhadap kandungan dadah haram dalam cecair badan yang dibekalkan oleh sebuah pelanggan perundangan. Bagaimanapun, banyak peralatan mereka selalu mengalami kerosakkan.
- (i) Perikan penggunaan gambarajah Ishikawa daripada kumpulan alat gemilang tujuh yang terdapat dalam proses kawalan statistik untuk anda mencari punca masalah ini.
 - (ii) Pengurus makmal ini membuat keputusan untuk menggantikan alat kromatografi prestasi tinggi (HPLC). Bincangkan beberapa faktor yang perlu beliau pertimbangkan dalam membantu beliau membuat keputusan tersebut.
- (10 markah)
- (b) Pensampelan penerimaan melibatkan penggunaan suatu pelan yang telah ditetapkan dahulu untuk memutuskan sama ada suatu kelompok barang memenuhi kriteria penerimaan yang ditakrifkan.
- (i) Huraikan penggunaan keluk ciri pengoperasian dalam pensampelan penerimaan.
 - (ii) Bandingkan pelan pensampelan tunggal dan dubel.
- (10 markah)
5. (a) Data di dalam jadual di bawah adalah data pemantauan harian kandungan plumbum (Pb) dalam air sungai yang diberikan dalam soalan 3a tetapi untuk hanya 13 hari. Data ini telah diproses untuk melukis carta CUSUM.
- (i) Jelaskan konsep carta CUSUM dan berikan kelebihanannya berbanding carta x-bar.
 - (ii) Isikan nilai-nilai kosong di dalam jadual tersebut.

(iii) Lukiskan carta CUSUM untuk proses analisis ini.

| Day | \bar{X} | Z_i | C_i |
|-----|-----------|-------|-------|
| 1 | 7.2 | 1.83 | 1.83 |
| 2 | 6.2 | | |
| 3 | 3.4 | -1.97 | 0.69 |
| 4 | 6.8 | 1.43 | 2.12 |
| 5 | 4.8 | -0.57 | 1.55 |
| 6 | 8.2 | | |
| 7 | 4.0 | -1.37 | 3.01 |
| 8 | 3.6 | -1.77 | 1.24 |
| 9 | 4.4 | -0.97 | 0.27 |
| 10 | 3.8 | | |
| 11 | 5.6 | 0.23 | -1.07 |
| 12 | 3.6 | -1.77 | -2.84 |
| 13 | 8.2 | 2.83 | -0.01 |

(12 markah)

(b) Bezakan di antara sebutan-sebutan ujian pengesahan di bawah.

- (i) Kespesifikan dan kepilihan
- (ii) Kebolehulangan dan keterulangan
- (iii) Kepersisan pertengahan dan kepersisan peralatan
- (iv) Sensitiviti dan kelinearan

(8 markah)

6. Faber et al. (Journal of Chromatography A, 897 (2000) 329–338) telah melaporkan penggunaan rekabentuk Plackett-Burman paras-dua untuk ujian ketegapan kaedah elektroforesis rerambut menggunakan teknik suntikan “short-end” bagi menilai kinetik pemelarutan kalsium akamprostat daripada tablet saduran enterik. Lapan faktor berpotensi kritis dipilih yang juga melibatkan tiga faktor dumi. Kesan yang telah dikira menggunakan rekabentuk N=12 diberikan di dalam jadual di bawah.
- (i) Guna plot separuh normal untuk mengenalpasti kesan ketara sekiranya ada dan faktor yang berkaitan dengannya daripada ujian ketegapan di atas.
 - (ii) Guna algorithma Dong untuk menganggarkan ralat dan kira ralat sisir (ME) pada $\alpha = 0.05$
 - (ii) Kira $E_{critical}$ berdasarkan ralat yang dianggarkan daripada faktor dumi dan kenalpastikan kesan ketara sekiranya ada bersama faktor-faktor yang berkaitan. Apakah terdapat sebarang anomali di antara keputusan (ii) dan (iii).

| Faktors | Kesan | Rankit |
|-----------------|---------|--------|
| [Akamprostat] | 0.041 | 0.06 |
| Voltan | -0.113 | 0.17 |
| [Sorbit] | -0.072 | 0.29 |
| Masa suntikan | -0.0613 | 0.41 |
| Jarak gelombang | 0.015 | 0.53 |
| Masa bilasan | 0.046 | 0.67 |
| Suhu | -0.121 | 0.81 |
| [Borat] | 0.461 | 0.98 |
| Dumi 1 | 0.0072 | 1.19 |
| Dumi2 | 0.034 | 1.45 |
| Dumi 3 | 0.022 | 1.91 |

(20 markah)

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7. (a) Mengikut garis panduan USEPA, kesemua makmal yang menggunakan relau grafit untuk penentuan plumbum dalam air minum wajib mencapai had pengesanan $1.5 \mu\text{g L}^{-1}$. Satu larutan stok mengandungi asid nitrik dan $5 \mu\text{g L}^{-1}$ plumbum telah disediakan. Keluk penentuan juga telah disediakan menggunakan 4 piawai pada 5, 10, 20, 50 $\mu\text{g L}^{-1}$ dan satu blank. Lapan alikuot sampel telah dianalisis mengikut satu kaedah tertentu. Keputusannya (dalam $\mu\text{g L}^{-1}$) diberikan di dalam jadual berikut.

| Nombor Sampel | Keputusan ($\mu\text{g L}^{-1}$) | Pemulihan (%) |
|---------------|------------------------------------|---------------|
| blank | -0.8 | NA |
| 1 | 4.9 | 98 |
| 2 | 4.7 | 94 |
| 3 | 4.6 | 92 |
| 4 | 4.5 | 90 |
| 5 | 4.7 | 94 |
| 6 | 4.8 | 96 |
| 7 | 4.8 | 96 |

- (i) Bezakan sebutan-sebutan had pengesanan kaedah (MDL) dan had kuantifikasi (LOQ).
- (ii) Tentukan had pengesanan kaedah dan had kuantifikasi (LOQ) untuk kaedah ini. Berikan verifikasi yang bersesuaian terhadap nilai MDL yang diperolehi.

(10 markah)

- (b) Piawai kualiti ISO telah diwujudkan untuk memberikan pengurusan kualiti bagi makmal dan industri di samping menjana hasil atau servis yang berkualiti

- (i) Apakah perbezaan utama di antara akreditasi dan pensijilan?
- (iii) Gariskan prosedur biasa dan persediaan untuk makmal anda mendapatkan akreditasi ISO 17025.

(10 markah)

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APPENDIXUseful Equations for the examination

$$M_j = \frac{0.6745(X_i - \bar{X})}{MAD}$$

$$z_i = \frac{(x_i - \bar{x})}{s}$$

Where

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

$$s_{ai} = \sqrt{\frac{n \sum (x_i - \bar{x})^2 (1 - u_i^2)^4}{(1 - u_i^2)(1 - 5u_i^2)}}$$

Where

$$u = \frac{x_i - \bar{x}}{9MAD}$$

$$R_{i+1} = \frac{|x^{(i)} - \bar{x}^{(i)}|}{s(i)}$$

$$Q_{exp} = X_q - X_n / w$$

$$t = \frac{|E_x|}{(SE)_e}$$

$$E_x = t_{critical} \text{ multiply } (SE_e)$$

$$(SE)_e \sqrt{\frac{s^2}{N/2} + \frac{s^2}{N/2}} = \sqrt{\frac{4S^2}{N}}$$

$$\sqrt{\left(\sum E^2_{error} / n_{error}\right)}$$

$$C_i = C_{i-1} + (\bar{x} - \mu_o)$$

$$s_o = 1.5 \text{ median } |E_i|$$

$$S_1 = \sqrt{m^{-1} \sum E_i^2}$$

$$ME = t_{(1-\alpha/2, df)} S_1$$

$$E_x = \frac{\sum Y^{(+)}}{N/2} - \frac{\sum Y^{(-)}}{N/2}$$

$$SE = \sqrt{\frac{\sum E^2_{emr}}{n_{emr}}}$$

$$\sigma_m = \frac{\sigma}{\sqrt{N}}$$

$$CL = \bar{x} \pm \frac{ts}{\sqrt{N}}$$

$$C = \frac{X_{(2)} - X_{(1)}}{X_{(n-1)} - X_{(1)}}$$

$$C = \frac{X_{(n)} - X_{(n-1)}}{X_{(n)} - X_{(2)}}$$

$$G_1 = \frac{|\bar{x} - x_i|}{s}$$

$$G_2 = \frac{x_n - x_1}{s}$$

$$G_3 = 1 - \left(\frac{(n-3) \times S_{n-2}^2}{(n-1)(s^2)} \right)$$

$$C_i^+ = \max[0, X_i - (\mu_o + K) + C_{i-1}^+]$$

$$C_i^- = \max[0, (\mu_o + K) - X_i + C_{i-1}^-]$$

$$C_n = \frac{\text{Suspected}(S^2)}{\sum_{i=1}^g Si^2}$$

$$\bar{n} = \frac{\sum_{i=1}^g n_i}{\delta}$$

$$t_{\text{obs}} = \frac{\bar{x}_1 - \bar{x}_2}{S_{\bar{x}}}$$

$$SE_{\text{pooled}} = \sqrt{s^2_{\text{pooled}} \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$$

$$S^2_{\text{pooled}} = \frac{SS_1 + SS_2}{n_1 + n_2 - 2}$$

$$t = \frac{\bar{d}}{\sqrt{s^2/n}}$$

| | | | |
|------------|-----------|--|-------|
| | df | ss | ms |
| Bef Group | I-1 | $\sum ni(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$ | ss/df |
| With Group | $n^* - I$ | $\sum \sum (y_{ij} - \bar{y}_{i\cdot})^2$ | ss/df |

$$\frac{|\bar{x}_1 - \bar{x}_2|}{\sqrt{MSW \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} > t_{\alpha/2}$$

$$b_1 = \frac{SP_{xy}}{SS_x}$$

$$S_{ylx} = \sqrt{\frac{SS_{\text{resid}}}{n-2}}$$

$$SE_{b1} = \sqrt{\frac{S^2_{y/x}}{SS_x}}$$

$$r = \frac{b_1 S_x}{S_r}, r^2 = \frac{SS_{\text{resid}}}{SS_y}$$

$$CV = 2^{(1-0.5 \log c)}$$

$$S_1 = \sqrt{m^{-1} \sum E_i^2}$$

$$ME = t_{(1-\alpha/2, df)} \cdot S_1$$

$$\sigma_{\bar{x}} = \frac{A_2 \bar{R}}{3} \quad s = \sigma_{\bar{x}} \sqrt{n} = \frac{\bar{R}}{d_2}$$

$$CL = \bar{x} \pm 3\sigma$$

$$Cp = \frac{USL - LCL}{6\sigma}$$

$$s_D = \sqrt{\frac{(m-1)s_C^2 + (n_1-1)s_{n_1}^2 + \dots + (n_{k-1}-1)s_{k-1}^2}{(m-1) + (n_1-1) + \dots + (n_{k-1}-1)}}$$

$$t_i = \frac{|\bar{X}_i - \bar{X}_C|}{s_D \sqrt{1/n_i + 1/n_C}}$$

$$F_{12} = \frac{(\bar{X}_1 - \bar{X}_2)^2}{MS_w \left(\frac{1}{n_1} + \frac{1}{n_2} \right) (K-1)}$$

$$S_{\text{obs}} = \frac{|\bar{X}_B - \bar{X}_A|}{SE_{\text{Scheffe - means}}}$$

$$SE_{Scheffe-subsets} = \sqrt{(s_w^2) \left(\sum \frac{c_i^2}{n_i} \right)}$$

$$SE_{Scheffe-means} = \sqrt{(s_w^2) \left(\frac{1}{n_A} + \frac{1}{n_B} \right)}$$

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Upper percentage points of Cochran's Test for homogeneity of Variance

| df for $\hat{\sigma}_j^2$ | α | $k = \text{number of variances}$ | | | | | | | | | | |
|---------------------------|----------|----------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| | | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 15 | 20 |
| 1 | .05 | .9985 | .9669 | .9065 | .8412 | .7808 | .7271 | .6798 | .6385 | .6020 | .4709 | .3894 |
| | .01 | .9999 | .9933 | .9676 | .9279 | .8828 | .8376 | .7945 | .7544 | .7175 | .5747 | .4799 |
| 2 | .05 | .9750 | .8709 | .7679 | .6838 | .6161 | .5612 | .5157 | .4775 | .4450 | .3346 | .2705 |
| | .01 | .9950 | .9423 | .8643 | .7885 | .7218 | .6644 | .6152 | .5727 | .5358 | .4069 | .3297 |
| 3 | .05 | .9392 | .7977 | .6841 | .5981 | .5321 | .4800 | .4377 | .4027 | .3733 | .2758 | .2205 |
| | .01 | .9794 | .8831 | .7814 | .6957 | .6258 | .5685 | .5209 | .4810 | .4469 | .3317 | .2654 |
| 4 | .05 | .9057 | .7457 | .6287 | .5441 | .4803 | .4307 | .3910 | .3584 | .3311 | .2419 | .1921 |
| | .01 | .9586 | .8335 | .7212 | .6329 | .5635 | .5080 | .4627 | .4251 | .3934 | .2882 | .2288 |
| 5 | .05 | .8772 | .7071 | .5895 | .5065 | .4447 | .3974 | .3595 | .3286 | .3029 | .2195 | .1735 |
| | .01 | .9373 | .7933 | .6761 | .5875 | .5195 | .4659 | .4226 | .3870 | .3572 | .2593 | .2048 |
| 6 | .05 | .8534 | .6771 | .5598 | .4783 | .4184 | .3726 | .3362 | .3067 | .2823 | .2034 | .1602 |
| | .01 | .9172 | .7606 | .6410 | .5531 | .4866 | .4347 | .3932 | .3592 | .3308 | .2386 | .1877 |
| 7 | .05 | .8332 | .6530 | .5365 | .4564 | .3980 | .3535 | .3185 | .2901 | .2666 | .1911 | .1501 |
| | .01 | .8988 | .7335 | .6129 | .5259 | .4608 | .4105 | .3704 | .3378 | .3106 | .2228 | .1748 |
| 8 | .05 | .8159 | .6333 | .5175 | .4387 | .3817 | .3384 | .3043 | .2768 | .2541 | .1815 | .1422 |
| | .01 | .8823 | .7107 | .5897 | .5037 | .4401 | .3911 | .3522 | .3207 | .2945 | .2104 | .1646 |
| 9 | .05 | .8010 | .6167 | .5017 | .4241 | .3682 | .3259 | .2926 | .2659 | .2439 | .1736 | .1357 |
| | .01 | .8674 | .6912 | .5702 | .4854 | .4229 | .3751 | .3373 | .3067 | .2813 | .2002 | .1567 |
| 16 | .05 | .7341 | .5466 | .4366 | .3645 | .3135 | .2756 | .2462 | .2226 | .2032 | .1429 | .1108 |
| | .01 | .7949 | .6059 | .4884 | .4094 | .3529 | .3105 | .2779 | .2514 | .2297 | .1612 | .1248 |
| 36 | .05 | .6602 | .4748 | .3720 | .3066 | .2612 | .2278 | .2022 | .1820 | .1655 | .1144 | .0879 |
| | .01 | .7067 | .5153 | .4057 | .3351 | .2858 | .2494 | .2214 | .1992 | .1811 | .1251 | .0960 |
| 144 | .05 | .5813 | .4031 | .3093 | .2513 | .2119 | .1833 | .1616 | .1446 | .1308 | .0889 | .0675 |
| | .01 | .6062 | .4230 | .3251 | .2644 | .2229 | .1929 | .1700 | .1521 | .1376 | .0934 | .0709 |

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Critical values for Dixon's Test

| Sample Size | For Averages | For Ranges | | Standard Deviation |
|-------------|----------------|----------------|----------------|--------------------|
| | | D ₃ | D ₄ | |
| n | A ₂ | D ₃ | D ₄ | d ₂ |
| 2 | 1.88 | 0 | 3.29 | 1.13 |
| 3 | 1.02 | 0 | 2.58 | 1.69 |
| 4 | .73 | 0 | 2.28 | 2.06 |
| 5 | .58 | 0 | 2.11 | 2.33 |
| 6 | .48 | 0 | 2.00 | 2.53 |
| 7 | .42 | 0 | 1.92 | 2.70 |

| | | | | |
|----|-----|-----|------|------|
| 8 | .37 | .14 | 1.87 | 2.85 |
| 9 | .34 | .18 | 1.82 | 2.97 |
| 10 | .31 | .31 | 1.78 | 3.08 |

Dunnets Table

| n | Level of Significance α | | |
|----|--------------------------------|-------|-------|
| | 0.10 | 0.05 | 0.01 |
| 3 | 0.886 | 0.941 | 0.988 |
| 4 | 0.679 | 0.765 | 0.889 |
| 5 | 0.557 | 0.642 | 0.780 |
| 6 | 0.482 | 0.560 | 0.698 |
| 7 | 0.434 | 0.507 | 0.637 |
| 8 | 0.479 | 0.554 | 0.683 |
| 9 | 0.441 | 0.512 | 0.635 |
| 10 | 0.409 | 0.477 | 0.597 |
| 11 | 0.517 | 0.576 | 0.679 |
| 12 | 0.490 | 0.546 | 0.642 |
| 13 | 0.467 | 0.521 | 0.615 |
| 14 | 0.492 | 0.546 | 0.641 |
| 15 | 0.472 | 0.525 | 0.616 |
| 16 | 0.454 | 0.507 | 0.595 |
| 17 | 0.438 | 0.490 | 0.577 |
| 18 | 0.424 | 0.475 | 0.561 |
| 19 | 0.412 | 0.462 | 0.547 |
| 20 | 0.401 | 0.450 | 0.535 |
| 21 | 0.391 | 0.440 | 0.524 |
| 22 | 0.382 | 0.430 | 0.514 |
| 23 | 0.374 | 0.421 | 0.505 |
| 24 | 0.367 | 0.413 | 0.497 |
| 25 | 0.360 | 0.406 | 0.489 |

Table of constants for Control Chart-X-bar

| Degrees of Freedom | α | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 | 14 | 16 |
|--------------------|------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| | | 2 | .05 .10 | 3.80 2.54 | 4.34 2.92 | 4.71 3.20 | 5.08 3.40 | 5.24 3.57 | 5.43 3.71 | 5.60 3.83 | 5.75 3.94 | 5.88 4.03 | 6.11 4.19 |
| 3 | .05 .10 | 2.94 2.13 | 3.28 2.41 | 3.52 2.61 | 3.70 2.76 | 3.85 2.87 | 3.97 2.97 | 4.08 3.06 | 4.17 3.13 | 4.25 3.20 | 4.39 3.31 | 4.51 3.41 | 4.61 3.49 |
| 4 | .05 .10 | 2.61 1.96 | 2.88 2.20 | 3.08 2.37 | 3.22 2.50 | 3.34 2.60 | 3.44 2.68 | 3.52 2.75 | 3.59 2.82 | 3.66 2.87 | 3.77 2.97 | 3.86 3.05 | 3.94 3.11 |
| 5 | .05 .10 | 2.44 1.87 | 2.68 2.09 | 2.85 2.24 | 2.98 2.36 | 3.08 2.45 | 3.16 2.53 | 3.24 2.59 | 3.30 2.65 | 3.36 2.70 | 3.45 2.78 | 3.53 2.86 | 3.60 2.92 |
| 6 | .05 .10 | 2.34 1.82 | 2.56 2.02 | 2.71 2.17 | 2.83 2.27 | 2.92 2.36 | 3.00 2.43 | 3.06 2.49 | 3.12 2.54 | 3.17 2.59 | 3.26 2.67 | 3.33 2.74 | 3.48 2.79 |
| 7 | .05 .10 | 2.27 1.78 | 2.48 1.98 | 2.82 2.11 | 2.73 2.22 | 2.81 2.30 | 2.89 2.37 | 2.95 2.42 | 3.00 2.47 | 3.05 2.52 | 3.13 2.59 | 3.20 2.66 | 3.26 2.71 |
| 8 | .05 .10 | 2.22 1.75 | 2.42 1.94 | 2.55 2.08 | 2.66 2.17 | 2.74 2.25 | 2.81 2.32 | 2.87 2.38 | 2.92 2.42 | 2.96 2.47 | 3.04 2.54 | 3.11 2.60 | 3.16 2.65 |
| 9 | .05 .10 | 2.18 1.73 | 2.37 1.92 | 2.50 2.05 | 2.60 2.14 | 2.68 2.22 | 2.75 2.28 | 2.81 2.34 | 2.86 2.39 | 2.90 2.43 | 2.97 2.50 | 3.04 2.56 | 3.09 2.61 |
| 10 | .05 .10 | 2.15 1.71 | 2.34 1.90 | 2.47 2.02 | 2.56 2.12 | 2.64 2.19 | 2.70 2.26 | 2.76 2.31 | 2.81 2.35 | 2.85 2.40 | 2.92 2.46 | 2.98 2.52 | 3.03 2.57 |
| 12 | .05 .10 | 2.11 1.69 | 2.29 1.87 | 2.41 1.99 | 2.50 2.08 | 2.58 2.16 | 2.64 2.22 | 2.69 2.27 | 2.74 2.31 | 2.78 2.35 | 2.84 2.42 | 2.90 2.47 | 2.95 2.52 |
| 16 | .05 .10 | 2.06 1.66 | 2.23 1.83 | 2.34 1.95 | 2.43 2.04 | 2.50 2.11 | 2.56 2.17 | 2.61 2.22 | 2.65 2.26 | 2.69 2.30 | 2.75 2.36 | 2.81 2.41 | 2.85 2.46 |

Table of constants for Control Chart-R-bar

| Sample size | D4 | D3 | DWLR | DWUR |
|-------------|------|------|------|------|
| 2 | 3.27 | 0 | 0.0 | 2.51 |
| 3 | 2.57 | 0 | 0.18 | 2.17 |
| 4 | 2.28 | 0 | 0.29 | 1.93 |
| 5 | 2.11 | 0 | 0.37 | 1.81 |
| 6 | 2.00 | 0 | 0.42 | 1.72 |
| 7 | 1.92 | 0.08 | 0.46 | 1.66 |
| | | | | |

Grubbs' critical value table:

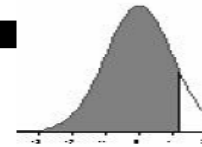
| N | 0.1 | 0.075 | 0.05 | 0.025 | 0.01 | | N | 0.1 | 0.075 | 0.05 | 0.025 | 0.01 |
|----|------|-------|------|-------|------|--|----|-----|-------|-------|-------|------|
| 3 | 1.15 | 1.15 | 1.15 | 1.15 | 1.15 | | 53 | 0 | 0 | 2.981 | 3.151 | 999 |
| 4 | 1.42 | 1.44 | 1.46 | 1.48 | 1.49 | | 54 | 0 | 0 | 2.988 | 3.158 | 999 |
| 5 | 1.6 | 1.64 | 1.67 | 1.71 | 1.75 | | 55 | 0 | 0 | 2.995 | 3.165 | 999 |
| 6 | 1.73 | 1.77 | 1.82 | 1.89 | 1.94 | | 56 | 0 | 0 | 3.002 | 3.172 | 999 |
| 7 | 1.83 | 1.88 | 1.94 | 2.02 | 2.1 | | 57 | 0 | 0 | 3.009 | 3.179 | 999 |
| 8 | 1.91 | 1.96 | 2.03 | 2.13 | 2.22 | | 58 | 0 | 0 | 3.016 | 3.186 | 999 |
| 9 | 1.98 | 2.04 | 2.11 | 2.21 | 2.32 | | 59 | 0 | 0 | 3.023 | 3.193 | 999 |
| 10 | 2.03 | 2.1 | 2.18 | 2.29 | 2.41 | | 60 | 0 | 0 | 3.03 | 3.2 | 999 |
| 11 | 2.09 | 2.14 | 2.23 | 2.36 | 2.48 | | 61 | 0 | 0 | 3.036 | 3.206 | 999 |
| 12 | 2.13 | 2.2 | 2.29 | 2.41 | 2.55 | | 62 | 0 | 0 | 3.042 | 3.212 | 999 |
| 13 | 2.17 | 2.24 | 2.33 | 2.46 | 2.61 | | 63 | 0 | 0 | 3.048 | 3.218 | 999 |
| 14 | 2.21 | 2.28 | 2.37 | 2.51 | 2.66 | | 64 | 0 | 0 | 3.054 | 3.224 | 999 |
| 15 | 2.25 | 2.32 | 2.41 | 2.55 | 2.71 | | 65 | 0 | 0 | 3.06 | 3.23 | 999 |
| 16 | 2.28 | 2.35 | 2.44 | 2.59 | 2.75 | | 66 | 0 | 0 | 3.066 | 3.236 | 999 |
| 17 | 2.31 | 2.38 | 2.47 | 2.62 | 2.79 | | 67 | 0 | 0 | 3.072 | 3.242 | 999 |
| 18 | 2.34 | 2.41 | 2.5 | 2.65 | 2.82 | | 68 | 0 | 0 | 3.078 | 3.248 | 999 |
| 19 | 2.36 | 2.44 | 2.53 | 2.68 | 2.85 | | 69 | 0 | 0 | 3.084 | 3.254 | 999 |
| 20 | 2.38 | 2.46 | 2.56 | 2.71 | 2.88 | | 70 | 0 | 0 | 3.09 | 3.26 | 999 |

Normal distribution table-Z table

| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 0.0 | 0.50000 | 0.50399 | 0.50798 | 0.51197 | 0.51595 | 0.51994 | 0.52392 | 0.52790 | 0.53188 | 0.53586 |
| 0.1 | 0.53983 | 0.54380 | 0.54776 | 0.55172 | 0.55567 | 0.55962 | 0.56358 | 0.56749 | 0.57142 | 0.57535 |
| 0.2 | 0.57926 | 0.58317 | 0.58706 | 0.59095 | 0.59483 | 0.59871 | 0.60257 | 0.60642 | 0.61026 | 0.61409 |
| 0.3 | 0.61791 | 0.62172 | 0.62552 | 0.62930 | 0.63307 | 0.63683 | 0.64058 | 0.64431 | 0.64803 | 0.65173 |
| 0.4 | 0.65542 | 0.65910 | 0.66276 | 0.66640 | 0.67003 | 0.67364 | 0.67724 | 0.68082 | 0.68439 | 0.68793 |
| 0.5 | 0.69146 | 0.69497 | 0.69847 | 0.70194 | 0.70540 | 0.70884 | 0.71228 | 0.71566 | 0.71904 | 0.72240 |
| 0.6 | 0.72575 | 0.72907 | 0.73237 | 0.73565 | 0.73891 | 0.74215 | 0.74537 | 0.74857 | 0.75175 | 0.75490 |
| 0.7 | 0.75804 | 0.76115 | 0.76424 | 0.76730 | 0.77035 | 0.77337 | 0.77637 | 0.77935 | 0.78230 | 0.78524 |
| 0.8 | 0.78814 | 0.79103 | 0.79389 | 0.79673 | 0.79955 | 0.80234 | 0.80511 | 0.80785 | 0.81057 | 0.81327 |
| 0.9 | 0.81594 | 0.81859 | 0.82121 | 0.82381 | 0.82639 | 0.82894 | 0.83147 | 0.83398 | 0.83646 | 0.83891 |
| 1.0 | 0.84134 | 0.84375 | 0.84614 | 0.84849 | 0.85083 | 0.85314 | 0.85543 | 0.85769 | 0.85993 | 0.86214 |
| 1.1 | 0.86433 | 0.86650 | 0.86864 | 0.87076 | 0.87286 | 0.87493 | 0.87698 | 0.87900 | 0.88100 | 0.88298 |
| 1.2 | 0.88493 | 0.88688 | 0.88877 | 0.89065 | 0.89251 | 0.89435 | 0.89617 | 0.89796 | 0.89973 | 0.90147 |
| 1.3 | 0.90320 | 0.90490 | 0.90658 | 0.90824 | 0.90988 | 0.91149 | 0.91308 | 0.91466 | 0.91621 | 0.91774 |
| 1.4 | 0.91924 | 0.92073 | 0.92220 | 0.92364 | 0.92507 | 0.92647 | 0.92785 | 0.92922 | 0.93056 | 0.93189 |
| 1.5 | 0.93319 | 0.93448 | 0.93574 | 0.93699 | 0.93822 | 0.93943 | 0.94062 | 0.94179 | 0.94295 | 0.94408 |
| 1.6 | 0.94520 | 0.94630 | 0.94738 | 0.94845 | 0.94950 | 0.95053 | 0.95154 | 0.95254 | 0.95352 | 0.95449 |
| 1.7 | 0.95543 | 0.95637 | 0.95728 | 0.95818 | 0.95907 | 0.95994 | 0.96080 | 0.96164 | 0.96246 | 0.96327 |
| 1.8 | 0.96407 | 0.96485 | 0.96562 | 0.96638 | 0.96712 | 0.96784 | 0.96856 | 0.96926 | 0.96995 | 0.97062 |
| 1.9 | 0.97128 | 0.97193 | 0.97257 | 0.97320 | 0.97381 | 0.97441 | 0.97500 | 0.97558 | 0.97615 | 0.97670 |
| 2.0 | 0.97725 | 0.97778 | 0.97831 | 0.97882 | 0.97932 | 0.97982 | 0.98030 | 0.98077 | 0.98124 | 0.98169 |
| 2.1 | 0.98214 | 0.98257 | 0.98300 | 0.98341 | 0.98382 | 0.98422 | 0.98461 | 0.98500 | 0.98537 | 0.98574 |
| 2.2 | 0.98610 | 0.98645 | 0.98679 | 0.98713 | 0.98745 | 0.98778 | 0.98809 | 0.98840 | 0.98870 | 0.98899 |
| 2.3 | 0.98928 | 0.98956 | 0.98983 | 0.99010 | 0.99036 | 0.99061 | 0.99086 | 0.99111 | 0.99134 | 0.99158 |
| 2.4 | 0.99180 | 0.99202 | 0.99224 | 0.99245 | 0.99266 | 0.99286 | 0.99305 | 0.99324 | 0.99343 | 0.99361 |
| 2.5 | 0.99379 | 0.99396 | 0.99413 | 0.99430 | 0.99446 | 0.99461 | 0.99477 | 0.99492 | 0.99506 | 0.99520 |
| 2.6 | 0.99534 | 0.99547 | 0.99560 | 0.99573 | 0.99585 | 0.99598 | 0.99609 | 0.99621 | 0.99632 | 0.99643 |
| 2.7 | 0.99653 | 0.99664 | 0.99674 | 0.99683 | 0.99693 | 0.99702 | 0.99711 | 0.99720 | 0.99728 | 0.99736 |
| 2.8 | 0.99744 | 0.99752 | 0.99760 | 0.99767 | 0.99774 | 0.99781 | 0.99788 | 0.99795 | 0.99801 | 0.99807 |
| 2.9 | 0.99813 | 0.99819 | 0.99825 | 0.99831 | 0.99836 | 0.99841 | 0.99846 | 0.99851 | 0.99856 | 0.99861 |
| 3.0 | 0.99865 | 0.99869 | 0.99874 | 0.99878 | 0.99882 | 0.99886 | 0.99889 | 0.99893 | 0.99896 | 0.99900 |
| 3.1 | 0.99903 | 0.99906 | 0.99910 | 0.99913 | 0.99916 | 0.99918 | 0.99921 | 0.99924 | 0.99926 | 0.99929 |
| 3.2 | 0.99931 | 0.99934 | 0.99936 | 0.99938 | 0.99940 | 0.99942 | 0.99944 | 0.99946 | 0.99948 | 0.99950 |
| 3.3 | 0.99952 | 0.99953 | 0.99955 | 0.99957 | 0.99958 | 0.99960 | 0.99961 | 0.99962 | 0.99964 | 0.99965 |
| 3.4 | 0.99966 | 0.99968 | 0.99969 | 0.99970 | 0.99971 | 0.99972 | 0.99973 | 0.99974 | 0.99975 | 0.99976 |
| 3.5 | 0.99977 | 0.99978 | 0.99978 | 0.99979 | 0.99980 | 0.99981 | 0.99981 | 0.99982 | 0.99983 | 0.99983 |
| 3.6 | 0.99984 | 0.99985 | 0.99985 | 0.99986 | 0.99986 | 0.99987 | 0.99987 | 0.99988 | 0.99988 | 0.99989 |
| 3.7 | 0.99989 | 0.99990 | 0.99990 | 0.99990 | 0.99991 | 0.99991 | 0.99992 | 0.99992 | 0.99992 | 0.99992 |
| 3.8 | 0.99993 | 0.99993 | 0.99993 | 0.99994 | 0.99994 | 0.99994 | 0.99994 | 0.99995 | 0.99995 | 0.99995 |
| 3.9 | 0.99995 | 0.99995 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99996 | 0.99997 | 0.99997 |
| 4.0 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99997 | 0.99998 | 0.99998 | 0.99998 | 0.99998 |

...15/-

Student's t-distribution table



| df | p | | | | | | | | | | |
|----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| | 0.75 | 0.80 | 0.85 | 0.90 | 0.95 | 0.975 | 0.980 | 0.990 | 0.995 | 0.9975 | 0.9990 |
| 1 | 1.0000 | 1.3764 | 1.9626 | 3.0777 | 6.3137 | 12.706 | 15.895 | 31.821 | 63.656 | 127.32 | 318.29 |
| 2 | 0.8165 | 1.0607 | 1.3862 | 1.8856 | 2.9200 | 4.3027 | 4.8487 | 6.9645 | 9.9250 | 14.089 | 22.329 |
| 3 | 0.7649 | 0.9785 | 1.2498 | 1.6377 | 2.3534 | 3.1824 | 3.4819 | 4.5407 | 5.8408 | 7.4532 | 10.214 |
| 4 | 0.7407 | 0.9410 | 1.1896 | 1.5332 | 2.1318 | 2.7765 | 2.9985 | 3.7469 | 4.6041 | 5.5975 | 7.1729 |
| 5 | 0.7267 | 0.9195 | 1.1558 | 1.4759 | 2.0150 | 2.5706 | 2.7565 | 3.3649 | 4.0321 | 4.7733 | 5.8935 |
| 6 | 0.7176 | 0.9057 | 1.1342 | 1.4398 | 1.9432 | 2.4469 | 2.6122 | 3.1427 | 3.7074 | 4.3168 | 5.2075 |
| 7 | 0.7111 | 0.8960 | 1.1192 | 1.4149 | 1.8946 | 2.3646 | 2.5168 | 2.9979 | 3.4995 | 4.0294 | 4.7853 |
| 8 | 0.7064 | 0.8889 | 1.1081 | 1.3968 | 1.8595 | 2.3060 | 2.4490 | 2.8965 | 3.3554 | 3.8325 | 4.5008 |
| 9 | 0.7027 | 0.8834 | 1.0997 | 1.3830 | 1.8331 | 2.2622 | 2.3984 | 2.8214 | 3.2498 | 3.6896 | 4.2969 |
| 10 | 0.6998 | 0.8791 | 1.0931 | 1.3722 | 1.8125 | 2.2281 | 2.3593 | 2.7638 | 3.1693 | 3.5814 | 4.1437 |
| 11 | 0.6974 | 0.8755 | 1.0877 | 1.3634 | 1.7959 | 2.2010 | 2.3281 | 2.7181 | 3.1058 | 3.4966 | 4.0248 |
| 12 | 0.6955 | 0.8726 | 1.0832 | 1.3562 | 1.7823 | 2.1788 | 2.3027 | 2.6810 | 3.0545 | 3.4284 | 3.9296 |
| 13 | 0.6938 | 0.8702 | 1.0795 | 1.3502 | 1.7709 | 2.1604 | 2.2816 | 2.6503 | 3.0123 | 3.3725 | 3.8520 |
| 14 | 0.6924 | 0.8681 | 1.0763 | 1.3450 | 1.7613 | 2.1448 | 2.2638 | 2.6245 | 2.9768 | 3.3257 | 3.7874 |
| 15 | 0.6912 | 0.8662 | 1.0735 | 1.3406 | 1.7531 | 2.1315 | 2.2485 | 2.6025 | 2.9467 | 3.2860 | 3.7329 |
| 16 | 0.6901 | 0.8647 | 1.0711 | 1.3368 | 1.7459 | 2.1199 | 2.2354 | 2.5835 | 2.9208 | 3.2520 | 3.6861 |
| 17 | 0.6892 | 0.8633 | 1.0690 | 1.3334 | 1.7396 | 2.1098 | 2.2238 | 2.5669 | 2.8982 | 3.2224 | 3.6458 |
| 18 | 0.6884 | 0.8620 | 1.0672 | 1.3304 | 1.7341 | 2.1009 | 2.2137 | 2.5524 | 2.8784 | 3.1966 | 3.6105 |
| 19 | 0.6876 | 0.8610 | 1.0655 | 1.3277 | 1.7291 | 2.0930 | 2.2047 | 2.5395 | 2.8609 | 3.1737 | 3.5793 |
| 20 | 0.6870 | 0.8600 | 1.0640 | 1.3253 | 1.7247 | 2.0860 | 2.1967 | 2.5280 | 2.8453 | 3.1534 | 3.5518 |
| 21 | 0.6864 | 0.8591 | 1.0627 | 1.3232 | 1.7207 | 2.0796 | 2.1894 | 2.5176 | 2.8314 | 3.1352 | 3.5271 |
| 22 | 0.6858 | 0.8583 | 1.0614 | 1.3212 | 1.7171 | 2.0739 | 2.1829 | 2.5083 | 2.8188 | 3.1188 | 3.5050 |
| 23 | 0.6853 | 0.8575 | 1.0603 | 1.3195 | 1.7139 | 2.0687 | 2.1770 | 2.4999 | 2.8073 | 3.1040 | 3.4850 |
| 24 | 0.6848 | 0.8569 | 1.0593 | 1.3178 | 1.7109 | 2.0639 | 2.1715 | 2.4922 | 2.7970 | 3.0905 | 3.4668 |
| 25 | 0.6844 | 0.8562 | 1.0584 | 1.3163 | 1.7081 | 2.0595 | 2.1666 | 2.4851 | 2.7874 | 3.0782 | 3.4502 |

Critical Values of the F Distribution
($\alpha = .05$)

| df within | df between | | | | | | | | | | |
|-----------|------------|------|------|------|------|------|------|------|------|------|----------|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 12 | 24 | ∞ |
| 5 | 6.61 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.68 | 4.53 | 4.37 |
| 6 | 5.99 | 5.14 | 4.76 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.00 | 3.84 | 3.67 |
| 7 | 5.59 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.57 | 3.41 | 3.23 |
| 8 | 5.32 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.28 | 3.12 | 2.93 |
| 9 | 5.12 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.07 | 2.90 | 2.71 |
| 10 | 4.96 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 2.91 | 2.74 | 2.54 |
| 11 | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.79 | 2.61 | 2.41 |
| 12 | 4.75 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.69 | 2.51 | 2.30 |
| 13 | 4.67 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.60 | 2.42 | 2.21 |
| 14 | 4.60 | 3.74 | 3.34 | 3.11 | 2.96 | 2.85 | 2.76 | 2.70 | 2.53 | 2.35 | 2.13 |
| 15 | 4.54 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.48 | 2.29 | 2.07 |
| 16 | 4.49 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.66 | 2.59 | 2.42 | 2.24 | 2.01 |
| 17 | 4.45 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.38 | 2.19 | 1.96 |
| 18 | 4.41 | 3.55 | 3.16 | 2.93 | 2.77 | 2.66 | 2.58 | 2.51 | 2.34 | 2.15 | 1.92 |
| 19 | 4.38 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.31 | 2.11 | 1.88 |
| 20 | 4.35 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.28 | 2.08 | 1.84 |
| 21 | 4.32 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.25 | 2.05 | 1.81 |
| 22 | 4.30 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.23 | 2.03 | 1.78 |
| 23 | 4.28 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.20 | 2.01 | 1.76 |
| 24 | 4.26 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.18 | 1.98 | 1.73 |
| 25 | 4.24 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.16 | 1.96 | 1.71 |
| 26 | 4.23 | 3.37 | 2.98 | 2.74 | 2.59 | 2.47 | 2.39 | 2.32 | 2.15 | 1.95 | 1.69 |
| 27 | 4.21 | 3.35 | 2.96 | 2.73 | 2.57 | 2.46 | 2.37 | 2.31 | 2.13 | 1.93 | 1.67 |
| 28 | 4.20 | 3.34 | 2.95 | 2.71 | 2.56 | 2.45 | 2.36 | 2.29 | 2.12 | 1.91 | 1.66 |
| 29 | 4.18 | 3.33 | 2.93 | 2.70 | 2.55 | 2.43 | 2.35 | 2.28 | 2.10 | 1.90 | 1.64 |
| 30 | 4.17 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.09 | 1.89 | 1.62 |
| 40 | 4.08 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.00 | 1.79 | 1.51 |
| 60 | 4.00 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 1.92 | 1.70 | 1.39 |
| 80 | 3.96 | 3.11 | 2.72 | 2.49 | 2.33 | 2.21 | 2.13 | 2.06 | 1.88 | 1.65 | 1.33 |
| 100 | 3.94 | 3.09 | 2.70 | 2.46 | 2.31 | 2.19 | 2.10 | 2.03 | 1.85 | 1.63 | 1.28 |
| 120 | 3.92 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.83 | 1.61 | 1.26 |
| ∞ | 3.84 | 3.00 | 2.61 | 2.37 | 2.22 | 2.10 | 2.01 | 1.94 | 1.75 | 1.52 | 1.00 |

Plackett-Burman Design For 11 Factors

| <i>Exp.</i> | <i>Factors</i> | | | | | | | | | | | <i>Response</i> |
|-------------|----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----------------|
| | A | B | C | D | E | F | G | H | I | J | K | |
| 1 | + | + | - | + | + | + | - | - | - | + | - | y1 |
| 2 | - | + | + | - | + | + | + | - | - | - | + | y2 |
| 3 | + | - | + | + | - | + | + | + | - | - | - | y3 |
| 4 | - | + | - | + | + | - | + | + | + | - | - | y4 |
| 5 | - | - | + | - | + | + | - | + | + | + | - | y5 |
| 6 | - | - | - | + | - | + | + | - | + | + | + | y6 |
| 7 | + | - | - | - | + | - | + | + | - | + | + | y7 |
| 8 | + | + | - | - | - | + | - | + | + | - | + | y8 |
| 9 | + | + | + | - | - | - | + | - | + | + | - | y9 |
| 10 | - | + | + | + | - | - | - | + | - | + | + | y10 |
| 11 | + | - | + | + | + | - | - | - | + | - | + | y11 |
| 12 | - | - | - | - | - | - | - | - | - | - | - | y12 |

Rankits to draw a half-normal plot for the most frequently used screening designs
(effect “1” indicates the smallest effect)

| Effect | Design size | | |
|--------|-------------|--------------|--------------|
| | <i>N</i> =8 | <i>N</i> =12 | <i>N</i> =16 |
| 1 | 0.09 | 0.06 | 0.04 |
| 2 | 0.27 | 0.17 | 0.12 |
| 3 | 0.46 | 0.29 | 0.21 |
| 4 | 0.66 | 0.41 | 0.29 |
| 5 | 0.90 | 0.53 | 0.38 |
| 6 | 1.21 | 0.67 | 0.47 |
| 7 | 1.71 | 0.81 | 0.57 |
| 8 | | 0.98 | 0.67 |
| 9 | | 1.19 | 0.78 |
| 10 | | 1.45 | 0.89 |
| 11 | | 1.91 | 1.02 |
| 12 | | | 1.18 |
| 13 | | | 1.36 |
| 14 | | | 1.61 |
| 15 | | | 2.04 |