Transient Natural Convection in a Rectangular Cavity Filled with Porous Medium Heated Discretely at Vertical Wall

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Abstract: Numerical analysis is presented for transient buoyancy-induced natural convection from discrete heating inside a two-dimensional rectangular enclosure filled with porous medium. The study has been made for Rayleigh numbers of 100, 1000 and 10000 with an aspect ratio of 2. The left vertical wall is heated discretely to a constant high temperature and the right wall is cooled to a constant low temperature. The remaining two walls are adiabatic. The finite volume method is used to solve the dimensionless governing equations. The distribution of stream function was observed and temperature distributions are presented at different timelines and analysed. In addition, the present results were also compared with those of previous researchers for isothermal walls and was found to be in good agreement.

Keywords: porous medium, natural convection, fluid flows, discrete heating, Nusselt number

1. INTRODUCTION

Buoyancy-driven fluid flow analysis in cavity that experiences convective heating or cooling at the surface are found in a wide variety of applications including lakes, geothermal reservoirs, underground water flows, solar collectors etc. An excellent introduction to the subject is given by Ostrach¹ with new developments in the area of natural convection heat transfer. Literatures concerning convective flow in porous media are abundant. Representative studies in this area may be found in recent books by Ingham and Pop,² Bejan and Kraus³ and Nield and Bejan.⁴

The research studies quoted pertained to steady state analysis employing numerical method, whereas the heat transfer through a porous medium is transient in nature. The implementation of transient analysis in numerical method has urged the researches to investigate the heat and flow

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pattern in the porous cavity. Weber⁵ obtained an analytical solution to the flow and heat transfer problem in the boundary layer regions of a porous enclosure using the Oseen linearisation procedure. Chan et al.⁶ and Bankwall⁷ have obtained numerical solution for the flow patterns and isotherms in a vertical enclosure at various instants of time. Later Singh and Misra⁸ investigated the unsteady two-dimensional free convective flow through a porous medium bound by infinite vertical plates. They considered the temperature of the plate is oscillating with time about a nonzero constant. However, Pop⁹ solved the problem of transient free convection in porous medium adjacent to a vertical semi-infinite flat plate with a steep increase in wall temperature and surface heat flux by an integral method. The paper proved that the boundary layer thickness grows continuously with time and approaches the steady state value asymptotically. Mehta and Sood¹⁰ used Karman-Pohlhausen integral method to study the problem of transient free convection flow about a non-isothermal vertical flat plate in a fluid saturated porous medium of variable permeability. But Saeid and Pop¹¹ highlighted the analysis on transient free convection in a two-dimensional square cavity filled with porous medium using finite volume numerical method. The analysis showed that the average Nusselt number is undershoot during the transient period for higher Rayleigh number.

From the literature review it is clear that most of the researchers employed numerical method to predict the behaviour of the porous medium. Their methods are well compared with the experimental results. Hence, the numerical methods become a foundation to researchers to justify the conformity of their predictions with the realistic situations. However, the study of transient natural convection in rectangular porous medium cavity with discrete heating is still lacking. Thus, finite volume method has been used in the present work to analyse the transient natural convection in porous medium induced by the discrete heating at different Rayleigh numbers.

2. EXPERIMENTAL

2.1 Basic Equation

A schematic diagram of a two-dimensional rectangular cavity is shown in Figure 1. It is assumed that the left vertical wall of the cavity is suddenly heated discretely at the middle to the constant temperature T_h and the right vertical wall is suddenly cooled to the constant temperature T_c , where $T_h > T_c$, by equal amount relative to an initial uniform temperature distribution, while the horizontal walls are maintained adiabatic as illustrated in Figure 1.



Figure 1: Schematic diagram of the physical system.

In the porous medium, Darcy's law is assumed to hold, and the fluid is assumed as a Boussinesq fluid. The viscous drag and inertia terms in the momentum equations are neglected, which are valid assumptions for low Darcy and particle Reynolds numbers. With these assumptions, the continuity, Darcy and energy equations for unsteady, two-dimensional flow in an isotropic and homogenous porous medium are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial^2 y}\right)$$
(2)

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$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + v\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial^2 y}\right) + g\beta(T - T_C)$$
(3)

$$\sigma \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$
(4)

With boundary conditions:

$$u (i, 0, \tau) = u (i, M, \tau) = u (0, j, \tau) = u (N, j, \tau) = 0$$

$$v (i, 0, \tau) = v (i, M, \tau) = v (0, j, \tau) = v (N, j, \tau) = 0$$

$$\frac{\partial T}{\partial y} (i, M, \tau) = 0, \quad 0 < i < N$$

$$T(0, j, \tau) = 0, \quad 0 < j < 30$$

$$T(0, j, \tau) = T_h, \quad 29 < j < 37$$

$$T(0, j, \tau) = 0, \quad 36 < j < M$$

$$T(N, j, \tau) = T_c, \quad 0 < j < M$$

Where i and j are the computational planes along x and y directions respectively and τ is the time in dimensionless form. N and M are the maximum grid number along i and j respectively. The velocity components u and v are the velocity in the x and y directions respectively; T denotes the temperature; v and α are kinematics viscosity and thermal diffusivity respectively; g is gravitational acceleration; β is 2 (T_h + T_c) P is the pressure, σ is the density and t is the time; T_h and T_c are the temperatures at hot bottom wall and cold vertical walls respectively; W and H are the cavity length and height respectively. The variables in equations 1 through 4 can be written in dimensionless forms as:

$$X = \frac{x}{W}, \quad Y = \frac{y}{H}, \quad \theta = \frac{T - T_h}{T_h - T_c}, \quad \Psi = \frac{\psi}{\alpha}, \quad \tau = \frac{\alpha t}{\sigma L^2}$$
(5)

The fluid mation is displayed using the stream function ψ obtained from velocity components u and v. The relationship between stream function ψ and velocity components for two dimensional flows¹¹ is:

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$$u = \frac{\partial \psi}{\partial y} \text{ and } v = \frac{\partial \varphi}{\partial x}$$
 (5)

Thus, by assuming the pressure difference is small and can be neglected, the governing equations above can be written in dimensionless forms as:

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra \frac{\partial \theta}{\partial X}$$
(6)

$$\frac{\partial\theta}{\partial\tau} + \frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial x} - \frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial y} = \frac{\partial^2\theta}{\partial X^2} + \frac{\partial^2\theta}{\partial Y^2}$$
(7)

The initial and boundary conditions have been set as:

$\Psi(i, j, 0) = 0$	$\theta(i, j, 0) = 0$
$\Psi(0, j, \tau) = 0$	$\theta(0, j, \tau) = 1$
$\Psi(1, j, \tau) = 0$	$\theta(1, j, \tau) = -0.5$
$\Psi(j,0,\tau) = 0$	$\partial \theta(j,0,\tau) / \partial Y = 0$
$\Psi(i,1,\tau)=0$	$\partial \theta(i,1,\tau) / \partial Y = 0$

The dimensionless numbers, local Nusselt (Nu), average Nusselt (Nu_{ave}) and Rayleigh (Ra) and aspect ratio (Ar) are calculated from:

$$Ra = \frac{g\beta K\Delta TL}{\nu\alpha}, \ Nu = -\left(\frac{\partial\theta}{\partial X}\right)_{X=0}, \ Nu_{ave} = \int_{0}^{1} Nu dy \text{ and } Ar = \frac{H}{W}$$
(8)

Where K is the permeability of the porous medium and ϵ is the void fraction which can be expressed as a function of bed porous and particle diameters as given by Nithiarasu et al.¹²:

$$K = \frac{\varepsilon^3 D_p^2}{150(1-\varepsilon)^2} \tag{9}$$

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2.2 Selection of Grid and Verification

The governing equations (6 and 7) are solved using the finite volume method with a fully implicit scheme. The power law scheme is used to solve the convection and diffusion terms as described by Patankar.¹³ The domain consists of non-uniform grid points in both X and Y directions. Various numbers of grids have been tested in order to get a good compromise between the precision of the results in term of average Nusselt number (on the hot face). All tests have been made for case of Ar = 1 and allowed to reach the steady state conditions in order to be confident with the results. The grid size of 65 x 65 was chosen due to a 2% different with Baytas¹⁴ and also because it consumed less computing time. The resulting algebraic equations were solved by line by line iteration using Tri-Diagonal Matrix Algorithm (TDMA).¹³ The parameters of convergence are fixed to 10^{-4} for both temperature and stream function. The effect grid numbers on average Nusselt number in the present analysis is show in Table 1.

Table 1: Effect of grid numbers on Nusselt number.

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No. of grids	Average Nu
45 imes 45	3.143
55×55	3.131
65 imes 65	3.123
85 imes 85	3.114

In the present solution, the verification of average Nusselt number values for Ra = 100, 1000 and 1000 have been considered and compared with the results obtained from Saeid and Pop,¹¹ Baytas¹⁴ and Bejan¹⁵ for isothermal temperature of both left and right walls at steady state condition. The top and bottom walls were set as adiabatic. These setting are used only for comparison purposes and also to validate the present results. The comparisons are shown in Table 2. The present results are found to be in good agreement with those of the previous workers. Therefore, these results provide great confidence to the accuracy of the present numerical model.

Table 2: Comparison of average Nusselt number with previous workers for Ar = 1.

Author	Ra = 100	Ra = 1000	Ra = 10000
Saeid and Pop ¹¹	3.002	13.726	43.953
Baytas ¹⁴	3.160	14.060	48.330
Bejan ¹⁵	4.200	15.800	50.800
Present result	3.123	13.964	48.197

3. **RESULTS AND DISCUSSION**

The isotherms and streamlines at different time steps ranging from $\tau = 0.0025$ to $\tau = 0.08$ are shown in Figure 2 for Ra = 1000. It can be seen that early in the transient, the isotherms are nearly parallel indicating conduction heat transfer near the right wall but at the left wall the fluid starts to rise up near the hot discrete wall (left). The fluid has fallen downward near the cooled right wall. Double vortices flow regions are observed from the streamline contours where a very weak secondary vortex developed close to the upper part of the hot wall and a primary circulation spinning the fluid towards the centre of the enclosure but is skewed due to the development of the secondary vortex [Fig. 2(a)]. Shortly after that, the fluid travels across the upper (or lower) half of the enclosure [Fig. 2(b)] and the secondary vortex grows bigger. The streamlines indicate an elongation of the re-circulation region of the flow along with a transition to the middle of the enclosure [Fig. 2(b)].

With increase of time ($\tau = 0.01$), the majority of fluid is rising up or falling down near the hot wall and near the cooled wall respectively [Fig. 2(c)] and the secondary vortex continuously growing. At the upper part of hot wall, the fluid temperature is observed higher compared to wall temperature, hence the local Nusselt number has negative magnitude which illustrates the heat is transferred into the wall [Fig. 3(b)].

Further, after a short time ($\tau = 0.02$), the primary vortex has been extended throughout the cavity and convection has became more important [Fig. 2(d)]. For $\tau > 0.04$ the flow is then going to attain the steady-state regime [Fig. 2(e)], similarly for $\tau = 0.08$ [Fig. 2(f)]. The isotherms and streamlines at $\tau = 0.08$ are presented in Figure 2(f), the development of the velocity and thermal boundary layers on the vertical walls of the cavity can be clearly observed from these figures that both vortices continuously grow to the steady-state thermal boundary layers. The development of the velocity and thermal boundary layers for Ra = 100 and 10000 are similar to those shown in Figure 2. The difference is that for low Rayleigh number condition the convection currents will be weaker which leads to a slower growth of boundary layer compared to that for high Rayleigh number condition. The stream lines and the isotherms for Ra = 100 and 10000 are not shown for brevity.



Figure 2: Isotherms (left) and streamlines (right) for Ra = 1000; (a) $\tau = 0.0025$, (b) $\tau = 0.005$, (c) $\tau = 0.01$, (d) $\tau = 0.02$, (e) $\tau = 0.04$ and (f) $\tau = 0.08$ (continued on next page).



Figure 2: (continued)



(e) $\tau = 0.04$



Figure 2: (continued)

The variation of unsteady local Nusselt number with time along the hot wall of the cavity at different positions Y is presented in Figure 3 for Ra = 100-10000. It can be seen that immediately after the process of heating starts

the value of the local Nusselt number increases drastically and this characteristic is observed in any heating system. Then, at small positions (Y = 0.0957), the local Nusselt number decreases for a short time followed by a constant value and then increases to reach the steady state condition. Figure 3(c) shows that this phenomenon will occur for the upper half also (Y = 0.5) for Ra = 1000. However, for Y = 0.859 and Ra = 100, 1000 and 10000 the local Nusselt number with negative magnitude decreases continuously with increase of time until it reaches its steady-state value. At this position, the results illustrate that the heat is transferred into the wall (left) since the fluid temperature is higher than the wall temperature.

Figure 4 shows that variation of the transient local Nusselt number is reflected on the average Nusselt number which is defined in equation 8. The results show the variation of the average Nusselt number with the dimensionless time for different Rayleigh numbers. The average Nusselt number showed a sudden increase during the transient period followed by a constant steady state value for Ra = 100–10000. It is also observed that the time required to reach the steady state (Nu becomes constant) is about $\tau = 0.04$ for all Rayleigh numbers as shown in Figures 3 and 4.



Figure 3: Variation of transient local Nusselt number with τ at different Rayleigh numbers: (a) Ra = 100, (b) Ra = 1000 and (c) Ra = 10000 (continued on next page).



Figure 3: (continued)



Figure 4: Variation of transient average Nusselt number with τ at different Rayleigh numbers.

4. CONCLUSION

The transient free convection in a two-dimensional rectangular cavity filled with a porous medium is considered in this paper. The flow is driven by considering the case when one of the vertical walls of the cavity is suddenly heated at a discrete location and the other vertical wall is suddenly cooled, while the horizontal walls are kept adiabatic. The non-dimensional forms of the continuity, Darcy and energy equations are solved numerically. The power-law scheme is used for the convection-diffusion formulation in the non-uniform grid in both horizontal and vertical directions. It is observed that during the transient period the average Nusselt number increases with time for Rayleigh numbers ranging from 100 to 10000. The results also show that the steady state is reached at $\tau = 0.04$ s for all Rayleigh numbers.

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