HIGHER CO-MOMENTS AND DOWNSIDE BETA IN ASSET PRICING

Imran Umer Chhapra* and Muhammad Kashif

Shaheed Zulfikar Ali Bhutto Institute of Science and Technology (SZABIST), Karachi, Sindh 75600, Pakistan

*Corresponding author: Imran_Js@yahoo.com

ABSTRACT

The Capital Asset Pricing Model (CAPM) assumes a linear relationship between an asset's return and financial market. However, empirical invalidity of linearity of returns has given birth to other CAPM models. Therefore, this study aims to examine the implication of preference by a risk-averse investor for higher moments and downside risk as investors are assumed to be prudent, temperate and cautious and prefer firms with negative coskewness, positive co-kurtosis, and downside risk as they yield higher risk premium. To empirically test these theoretical assumptions data of all 901 firms (listed and delisted) in Pakistan Stock Exchange (PSX) from 2000 to 2016 have been used. Decile portfolios are constructed for cross-sectional and time series analysis. Generalized Method of Moments (GMM) and Wald Test are applied to check the robustness of results. The results indicate that co-skewness, co-kurtosis and downside beta are important risk factors but only downside beta is genuinely priced over and above what co-variance risk can explain and CAPM does not significantly capture market risk premium indicating the existence of other risk measures in PSX. The findings can help investors in formulating investment strategies for constructing well-diversified and efficient portfolios and can enable firm managers to take appropriate capital budgeting decisions by appropriately costing equities.

Keywords: Asymmetries, higher moments, downside risk, asset pricing, CAPM

Publication date: 15 August 2019

To link to this article: https://doi.org/10.21315/aamjaf2019.15.1.6

© Asian Academy of Management and Penerbit Universiti Sains Malaysia, 2019. This work is licensed under the terms of the Creative Commons Attribution (CC BY) (http://creativecommons. org/licenses/by/4.0/).

To cite this article: Chhapra, I. U., & Kashif, M. (2019). Higher co-moments and downside beta in asset pricing. *Asian Academy of Management Journal of Accounting and Finance*, *15*(1), 129–155. https://doi.org/10.21315/aamjaf2019.15.1.6

INTRODUCTION

Pricing of a financial asset especially determination of the price of a risky financial asset is the most critical question in financial economics. It is in the field of asset pricing theory that has provided the answer to this critical question and is a dominant theme in financial economics. Hence, this has enticed the consideration of many investigators and academicians in finance plus a huge number of research publications have been penned on asset pricing (Dempsey, 2013). All the research studies have a considerably important question which is commonly asked: how do people distribute infrequent resources through a price system that is grounded on the estimation of risky assets and their expected returns? (Copeland, Weston, & Shastri, 2005). The partial answer to the above question was given by Markowitz (1952) in a groundbreaking research paper in which he describes how to select a portfolio using the Mean-Variance (MV) Framework¹. In MV framework, it is assumed that investor's preferences are defined with respect to the mean and variance of an asset return. It is assumed that asset returns are random variables and only the financial aspects of the portfolio (i.e. risk and return) affects the investor's investment decision and nothing else does (Vasant, Irgolic, Rajaratnam, & Kruger, 2014).

Moreover, it explains that investor selects the efficient portfolio² and algorithms were also developed by Markowitz for the systematic selection of efficient portfolios. He presented three building blocks that became the basis for the development of MPT that are; firstly, investors invest for only one-period holding return (R_i)³. Secondly; it is assumed that portfolio return's parameters are normally distributed because the parameters of asset returns are also normally distributed. Lastly, the investors' preferences for asset return are explained in terms of two moments i.e. μ_i (mean) and δ^2 (variance)⁴. Hence, Markowitz's works were so fundamental that his study and ideas permitted researchers to express a portfolio's risk in a quantifiable fashion which was not expressed by researchers earlier (Grauer, 2002).

Based on the MV framework, Sharpe (1964) and Lintner (1965) developed the most commonly used financial model known as the Capital Asset Pricing Model (CAPM). The CAPM model assumes that a linear relationship exists between the asset's expected return and the financial market. It is summarised in a single parameter and the systematic covariance (beta) risk is constant over time (Bajpai & Sharmab, 2015). The validity of this asset pricing model depends on two restrictive assumptions, namely that the utility function of an investor is quadratic and returns on assets are normally distributed. However, empirical evidence presented by Kraus and Litzenberger (1976), Fang and Lai (1997), Hwang and Satchell (1999), Fama and French (2004), Mergner and Bulla (2008), Choudhry and Wu (2009), Da, Guo, and Jagannathan (2012) and Schneider, Wagner, and Zechner (2016) suggest that the traditional CAPM with constant systematic covariance risk may be misleading and insufficient to characterize asset returns.

The above discussed empirical invalidity of linearity (symmetricity) of stock return in MV framework has given birth to Higher Moment CAPM Models (Kraus & Litzenberger, 1976; Fang & Lai, 1997; Hwang & Satchell, 1999; Harvey & Siddique, 2000; Dittmar, 2002; Mergner & Bulla, 2008; Choudhry & Wu, 2009). It has been concluded in various previous studies that returns on assets are non-normally distributed (asymmetric distribution) which means that distribution is skewed to the right or the left and has fatter tails. When a return is non-normal distributed, its tails on the ends include higher negative or positive returns which mean there is a higher than normal probability of large positive and negative returns. Similarly, kurtosis refers to the extent to which the distribution tends to have comparatively large frequencies around the midpoint (moderate loss probability) and in the tail of return distribution (large loss probability). Excess kurtosis indicates risk enhancement or risk reduction of a return distribution as it depends on the tradeoff between the fatness at the middle and at the tail of the distribution. Normally, excess kurtosis means the higher probability of extreme frequencies occurring in the tail of the return distribution. This incorporates one more assumption into the mean-variance framework i.e. disutility from capital depreciation is greater than utility from capital appreciation which means losses weigh stronger than profits (Kahneman & Tversky, 1979). A similar idea was given by Libby and Fishburn (1977) in their study that deviations below the mean value greatly affect the investor than the deviations above the mean value highlighting the concept of downside risk. Hence, the above phenomena have initiated the integration of higher-moments and downside beta in the traditional capital asset pricing model.

In Pakistan, various researchers such as Javid and Ahmad (2008), Javid and Ahmad (2011), Tahir, Abbas, Sargana, Ayub and Saeed (2013), Ayub, Shah and Abbas (2015) and Rashid and Hamid (2015) have tested higher co-moment and downside beta CAPM but their analysis is based on dataset of companies from selected sectors which result in a common problem of survivorship bias in portfolio management (see: Nagel, 2001). However, this paper provides more intensive results as it includes all companies of PSX. The key findings of this study explain that DSB is efficiently priced and CSK and CKT fail to yield abnormal average returns and cannot be considered as an additional risk source that is priced in PSX.

LITERATURE REVIEW

In financial literature, the most commonly and widely used asset pricing model based on asset pricing theory is Two-Moment or Traditional CAPM. However, many researchers in the past have criticized this model as misleading and insufficient to adequately characterize the equity market's returns. This may partly be a consequence of its restrictive assumptions. Therefore, existing literature proposed possible extensions by relaxing a few assumptions underlying the Two-Moment CAPM.

One extension of the Two-Moment CAPM is intended to answer criticism of the existence of a riskless asset. Black (1972) derived a Zero-Beta Model demonstrating that the results of the Two-Moment CAPM do not, in fact, require the existence of a riskless asset. Another extension was given by Merton (1973), who developed a multi-period model which is called Inter-temporal CAPM (ICAPM). ICAPM deals with the criticism of single time-period assumption of CAPM, in which investors maximise their portfolio at the end of the current period so that there does not exist any opportunity for investors to restore their portfolios repeatedly over time. The next extension was given by Breeden (1979) which relaxed the assumption that the market portfolio is not observable. Hence, Breeden (1979) developed Consumption CAPM (CCAPM) by using the consumption growth rate rather than the market portfolio's returns while explaining asset returns. Furthermore, Roll (1977) presented an Arbitrage Pricing Theory (APT) without a market portfolio return and showed that macroeconomic factors affect equity market returns. Another extension of Two-Moment CAPM deals with the relaxation of the assumption that investors have the same expected distribution of assets' returns in equity market which was developed by Bollerslev, Engle, and Wooldridge (1988).

Another aspect of capital assets pricing theory that has developed over a period of time is the development of multi-factor models. For example, one extension is by Banz (1981) in which he included size as one factor in the traditional CAPM. The results of the research showed that market capitalisation⁵ has an inverse relationship with returns and cross-sectional deviations of stock returns are explained comparatively much better than the beta of a stock. A further extension allows for the size and value of the assets which cannot be explained by the Two-Moment CAPM while explaining asset returns. Investors assume that the size and value of assets can affect the expected return of assets, so the risk level of assets can change in terms of their size and value. Hence, Fama and French (1993) derived the Fama-French model, extending the Two-Moment CAPM to fit additional factors such as size and value factors. Since then, other authors have also added additional factors. All of these led to a better description of the asset returns. The majority of these extensions, however, lacked simple interpretations in terms of risk.

Based on the above discussion, it may be concluded that traditional CAPM along with its extensions have a sound theoretical background but cannot be practically validated. Furthermore, the literature suggests that the majority of investors in the market are believed to be risk-averse, prudent, temperate and cautious as they prefer firms with negative co-skewness, positive co-kurtosis and downside risk yield over the firms that have positive co-skewness, negative co-kurtosis and upside risk yield. Therefore, this study criticizes the simple assumptions of normally distributed asset returns and quadratic utility as well as linear relationships between asset and market returns and follows the literature where, for example, Kraus and Litzenberger (1976), Fang and Lai (1997) and Hwang and Satchell (1999) derived Higher-Moment CAPMs or Roy (1952), Kahneman and Tversky (1979) and Estrada (2002) derived DCAPM, which extend the Two-Moment CAPM by including higher moments and downside beta respectively. These extensions are derived from a single factor (market portfolio return) instead of developing new factors. Hence, these models provide a simple interpretation in terms of risk, comparative to the other Two-Moment CAPM extensions, and are also easier to put into practice.

Higher-Moment CAPM

According to the literature, many authors have identified the existence of skewness and kurtosis in financial data (Kendall & Hill, 1953; Mandelbrot, 1963; Fama, 1965). As identified earlier that stock returns are non-normally distributed so skewness and kurtosis play a critical role in the determination of stock returns. Scott and Horvath (1980) suggested that investors are normally risk-averse, that is they like negative skewness and excess kurtosis. On the contrary, Arditti (1967) and Tufano (2008) showed that investors strongly prefer products with positively skewed returns.

Furthermore, one more assumption of CAPM is that investors' preferences are quadratic in nature but a quadratic utility function fails to identify investors' preferences and depicts Increasing Absolute Risk Aversion (IARA)⁶. Hence, this motivates researchers to incorporate higher moments in CAPM. It should be further noted that a basic requirement of a utility function is monotonicity. As satiation in wealth is an implausible property of quadratic utility function so when satiation is coupled with absolute risk aversion, it makes mean-variance models unrealistic for measuring the appropriate risk (see: Collins, & Gbur, 1991). As discussed above, that IARA fails to explain investor behaviour appropriately so researchers prefer Constant Relative Risk Aversion (CRRA)⁷ utility function (Campbell & Viceira, 2002; Kostakis, Muhammad, & Siganos, 2012). This function actually embeds aversion to negative skewness and excess kurtosis with an increase in Relative Risk Aversion (RRA) as an important feature of the CRRA for investor's preferences (Scott & Horvath, 1980). In addition to above discussion Kimball (1990), Kimball (1992) and Gollier and Pratt (1996) in their studies showed a utility function that incorporates investor's preference for positive skewness and investor's aversion to excess kurtosis. The former has been termed as "Prudence" whereas latter has been called "Temperance". They explained that if the market has a positive skewness of return then a prudent investor will select an asset with positive co-skewness whereas if the probability of extreme returns jointly occurring in an asset and market then a temperate investor will choose an asset with small co-kurtosis.

According to the literature, the higher-moment CAPMs capture coskewness (systematic skewness) and co-kurtosis (systematic kurtosis) in the distributions of financial data (Fama, 1965). The theory of these was developed by Kraus and Litzenberger (1976), Fang and Lai (1997) and Hwang and Satchell (1999). Authors empirically investigated the necessity for more complicated models by fitting Higher-Order Data Generating Processes (DGPs) to financial data; namely the Quadratic and Cubic Market Models. They proposed several formulations of higher order DGPs with the intention of successfully illustrating the link between higher order DGPs and their equivalent higher-moment CAPMs. For example, Barone-Adesi, (1985) proposed the Quadratic Market Model to be consistent with the three-moment CAPM that also captures co-skewness. Fang and Lai (1997), Hwang and Satchell (1999), Christie-David and Chaudhry (2001). Ranaldo and Favre (2005) and Galagedera and Jaapar (2009) proposed the Cubic Market Model to be consistent with the four-moment CAPM which captures both co-skewness and co-kurtosis to elucidate time series returns for various sets of financial data

Downside Beta CAPM (DCAPM)

As discussed earlier Markowitz elucidates that by calculating mean and variance of the expected return an investor can easily measure the risk and return of an investment. He further explains that variance reflects variation both above and below mean which contributes equally to the total risk as perceived by any investor. However, Prospects Theory's S-shaped utility function propounded by Kahneman and Tversky (1979) which is consistent with the study conducted by Libby and Fishburn (1977) in which the latter suggested that deviations which are below the mean affect the investor greater than the deviations above the mean thus displaying a behaviour of downside risk aversion. Further, it has been explained that these utility functions are based on the assumption that investors weigh losses more heavily than gains. Conclusively, investors aren't "risk averse" they are "loss averse." Initially, Markowitz also suggested and convinced that downside risk (semi-variance) is more appropriate to measure investor risk as compared to variance but due non-availability of advanced statistical tools and resources⁸ he stays with variance and discarded downside risk. However, he recognises the importance of downside risk (Markowitz, 1959).

The debate of downside risk was started by Roy (1952). In his study, he suggested that investors are "Cautious" as they care for disaster and safety from unforeseen events. Hence, he introduces *Safety-First Rule* (SF Rule). SF rule explains the creation of a portfolio that is based on the minimum level of portfolio returns, called the minimum acceptable return. In this way, the investor mitigates the risk of not achieving his investment objectives (Roy, 1952). However, Markowitz's MV framework and Roy's SF rule both consider normality of distribution which was rejected by many researchers like Kraus and Litzenberger (1976), Kon (1984), Fang and Lai (1997), Hwang and Satchell (1999), Fama and French (2004), Mergner and Bulla (2008), Choudhry and Wu (2009), Da et al. (2012), and Schneider et al. (2016). Hence, departure from normality forced the use of any other framework than MV framework to estimate the expected utility for an investor (Chunhachinda, Dandapani, Hamid, & Prakash, 1997; Athayde & Flores, 2004; Jondeau, & Rockinger, 2006).

Moreover, as suggested by Libby and Fishburn (1977) that investor gives more weight to downside risk than upside risk contradicts CAPM assumption of equal weightage to both type of risk by the investor (Kahneman & Tversky, 1979; Estrada, 2002). Estrada (2002) further argues that empirical evidence contradicts the underlying requirements of normal distribution and symmetry of returns, therefore, the variance is not considered to be a good measure of risk. Ang, Xing and Chen (2006) showed that downside beta is a better predictor of future outcome than MV framework whereas; Post and Vilet (2004) proved that DCAPM surpasses CAPM. Hence, many researcher like Quirk and Saposnik (1962), Mao (1970), Klemkosky (1973), Ang and Chua (1979), Grootveld and Hallerbach (1999), Harvey and Siddique (2000), Balzer (2001), Estrada (2002), Post and Vilet (2004), and Estrada and Serra (2005) concluded that downside beta is the best measure of risk among various risk measures in asset pricing models. Similarly, for calculating the cost of equity, Foong and Goh (2012) compared various CAPM measures for an emerging market and concluded that downside beta is the most relevant measure. Recently, many researches have

tested DCAPM in asset pricing like Alles and Murray (2017) tested downside, upside beta and co-skewness in the Australian equities market and found that they are priced in Australian stock market, however, downside and upside beta are not related to each other. In Pakistan, Raza (2018) and Rashid and Mehmood (2018) conducted researches on Pakistan stock market and financial institutions and provide evidence of a positive and statistically significant risk-return relationship. Similarly, Rashid and Hamid (2015) tested different DCAPMs⁹ to assess which downside beta better explain expected returns and found positive risk premium for downside beta of Bawa and Lindenberg (1977) and Harlow and Rao (1989) but positive and negative risk premium of Estrada (2002) downside beta in different sub-periods.

Concluding Remarks

Many researchers in the past have tried to evaluate this phenomenon of nonnormality and non-linearity in stock returns in both developed and emerging markets. Hence, for economic growth, stability, and prosperity, the stable equity market is very crucial for any country including Pakistan and stability in equity market can only be achieved through better understanding of risk and return dynamics of a stock market. The contribution of the study has threefold; first, it contributes to the existing literature by providing empirical evidence on both higher moment CAPM and downside beta CAPM in a volatile emerging market like Pakistan. Second, it examines the importance of the integration of higher moments and downside risk in asset pricing models as it assesses the existence of risk-averse, prudent, temperate and cautious investors in Pakistan Stock Exchange (PSX) by taking all listed and delisted firms in PSX to avoid survivorship bias and how they behave when firm share shows co-skewness, co-kurtosis, and downside risk. Third, it contributes to the debate on whether the cross-sectional asset returns are explained by systematic higher order co-moments and downside beta in the PSX because it will provide insight regarding the existence of additional risk factors that could improve the explanatory power of CAPM.

RESEARCH METHODOLOGY

Individual stocks data of all listed and delisted companies from 2000 to 2016 is collected from Pakistan Stock Exchange (PSX) and Thomson Reuters data stream to calculate monthly discrete asset returns and market returns. Furthermore, to avoid survivorship bias¹⁰ all listed and delisted companies are included in the dataset. Nagel (2001) considers survivorship bias to be a serious problem in stock return predictability studies because portfolios constructed on the data

with inherent ex-post selection bias do not represent trading strategies that are replicable ex-ante. Due to the inclusion of delisted firms in the sample, particular attention is paid to the reasons for a firm's delisting. Dimson, Nagel and Quigley (2003) and Soares and Stark (2009) are followed in correcting the delisting bias of Shumway (1997) by setting the stock return in the delisting month equal to -100%. Furthermore, the unit trust, investment trust, and ADRs are excluded. We have utilised both financial and non-financial firms as the sorting criteria are based on market information and not on accounting information (see: Kostakis et al., 2012). For CSK and CKT we followed Harvey and Siddique (2000) and DSB we used Estrada (2002)¹¹ approach over Bawa and Lindenberg (1977) and Harlow and Rao (1989) as it will generate correct and unbiased estimation of downside beta by using time series regression through origin (Rashid & Hamid, 2015).

Excess returns are calculated using 36-months rolling-windows regression of excess return $(R_{it} - R_i^f)$ and market excess return $(R_{mt} - R_i^f)$ and the corresponding residuals $\varepsilon_{i,t}$ are extracted:

$$(R_{it} - R_t^f) = \alpha_i + \beta_{iMKT} (R_{mt} - R_t^f) + \varepsilon_{it}$$
(1)

From the above regression model, the residuals $\varepsilon_{i,t}$ are orthogonal to the excess market returns. Therefore, these residuals are net of the covariance (beta) risk. However, they still incorporate co-skewness, co-kurtosis risk and downside beta and hence at, it is possible to obtain a measure of standardised CSK, CKT and DSB of each share's returns with the market returns over the period of *t*-36 to *t* via

$$CSK_{i} = \frac{E[\boldsymbol{\varepsilon}_{i,t+1} \boldsymbol{\varepsilon}_{M,t+1}^{2}]}{\sqrt{E[\boldsymbol{\varepsilon}_{i,t+1}^{2}]}E[\boldsymbol{\varepsilon}_{M,t+1}^{2}]}$$
(2)

and

$$CKT_{i} = \frac{E[\boldsymbol{\varepsilon}_{i,t+1}\boldsymbol{\varepsilon}_{M,t+1}^{3}]}{\sqrt{E[\boldsymbol{\varepsilon}_{i,t+1}^{2}]}(E[\boldsymbol{\varepsilon}_{M,t+1}^{3}])}$$
(3)

and

$$DSB_{im}^{-E} = \frac{E[\min(R_i - \mu_{i,0})\min(R_m - \mu_{m,0})]}{E[(\min(R_m - \mu_{m,0}))]^2}$$
(4)

where $(\varepsilon_{i,t})$ are the residuals from Equation (1). Similarly $(\varepsilon_{M,t})$ are the market residuals. Market residuals $(\varepsilon_{M,t})$ are the deviation of the excess market returns in month *t* from the average value over the corresponding window of observations *t*-36 to *t*. The advantage of deriving CSK and CKT from the residuals $(\varepsilon_{i,t})$ is that there is no dependency on the market returns; hence, they

are orthogonal to the market returns. These estimates give the account of the market beta in the similar way as the standard CAPM. Thus, CSK, CKT and DSB capture the excess return from every possible strategy that loads the CSK, CKT and DSB and represent the contribution of a security to the skewness, kurtosis and downside beta of the broader portfolio respectively. Having estimated the value of CSK, CKT and DSB for each share *i* at each month *t*, the decile portfolios are constructed based on these estimates as they provide more segmentation of sorting criteria (see: Fama, & French, 1993) as compare to percentiles portfolios¹² which help us in constructing more diversified and efficient portfolios.

At the end of the month, stocks are sorted separately according to their CSK, CKT and DSB measures in that month into ten portfolios. Portfolio 1 (P1) includes stocks with the lowest values of CSK, CKT, and DSB measures, while portfolio 10 (P10) contains stocks with the highest values. Portfolio returns are estimated monthly (i.e. post-ranking returns). Then both the equal and value-weighted portfolio returns in excess of the risk-free rate (6 months T-bills) are calculated.

STATISTICAL ANALYSIS

Descriptive Statistics

Descriptive statistics of equally weighted and value weighted decile portfolios constructed on the basis of co-skewness (CSK) (Table 1), co-kurtosis (CKT) (Table 2), and downside beta (DSB) (Table 3) measures are presented here. P1 portfolios are the portfolios with the lowest (negative) value of CSK, CKT, and DSB and P10 portfolios include the highest (positive) values of CSK, CKT, and DSB. The *t*-test is applied to the differences between the highest decile (P10) portfolio and lowest decile (P1) portfolio to check whether extreme portfolios behave differently or not. The results of value- and equally-weighted returns of portfolios sorted on the basis of CSK, CKT and DSB show a monotonically increasing pattern of returns and significant variation across all decile portfolio indicating the importance of CSK, CKT, and DSB as a sorting criterion.

Table 1 shows that P1 (negatively coskewed portfolio) contains stocks with a relatively lower market value in comparison to other portfolios (P2 to P10) and the portfolios beta depicts that P10 (positively coskewed portfolio) contains shares with relatively higher betas than there counterpart portfolios (P1 to P9). According to the mean-variance framework, the average return of P10 should yield a higher return than the average return of P1. The result of CSK portfolios shows that positively coskewed portfolio (P10) have a higher

average return than negatively coskewed portfolio (P1). The spread (P1–P10) of equally weighted and value-weighted returns is equal to -0.027% and -0.067% respectively but statistically insignificant. Our results contradict the findings of Harvey and Siddique (2000) and fail to prove that risk premium is required by prudent investors to invest in shares with negatively skewed returns in PSX.

Table 2 shows that positively co-kurtosis portfolio (P10) has shares with higher market value and lower CAPM beta than other corresponding portfolios (P1 to P9) implicating that P1 will yield higher average return than P10 as per CAPM. However, the result is indicating the higher return of P10 than P1 portfolio. Thus, it can be concluded that investors demand a risk premium to invest in positive co-kurtosis shares in PSX. Similar results were given by Harvey and Siddique (2000) and Galagedera, Maharaj and Brooks (2008) which concluded that investors preferred stocks with positive co-skewness and negative co-kurtosis otherwise they require a premium for investing and holding shares with negative co-skewness and positive co-kurtosis. Moreover, the decile portfolios based on DSB (Table 3) clearly shows that a highest DSB portfolio (P10) yield higher average return than lowest DSB portfolio (P1) and the spread (P10-P1) of equal and value-weighted portfolio are significant as compare to portfolios based on CSK and CKT indicating that these portfolios significantly better explain the risk and return relationship as compared to their counterpart part portfolios on CSK and CKT. Our results are consistent with Estrada (2002), Post and Vilet (2004), Ang et al. (2006) and Tahir et al. (2013). To conclude it can be said that in Pakistan, DSB is an important risk factor that needs to be considered at the time of portfolio analysis.

	ΡΙ	P2	P3	P4	P5	P6	P7	P8	$\mathbf{P9}$	P10	P1-P10	t-test
EW Excess Returns (% p.a)	0.131	0.192	0.420		0.292	0.358 0.292 0.640 0.096 0.414 0.066 0.158	0.096	0.414	0.066	0.158	-0.027	-0.320
VW Excess Returns (% p.a)	0.182	0.210	0.259	0.285	0.213	0.213 0.318 0.204 0.259	0.204	0.259	0.175	0.249	-0.067	-0.710
MV (Rs. tn)	2.830	2.880	2.530	2.950	4.250	2.950 4.250 3.410 3.460 4.160 4.350	3.460	4.160	4.350	6.100	-3.27	-6.297
CAPM β	-0.110	-0.110 0.060 -0.130 -0.010 0.020 0.150 0.100 0.030 0.010 0.070	-0.130	-0.010	0.020	0.150	0.100	0.030	0.010	0.070	-0.18	13.717
The above table presents characteristics of decile portfolios based on co-skewness from January 2000 to December 2016. All stocks listed in PSX are sorted in ascending of the tat month (i) according to the CSK values and are assigned to ten different portfolios (P1 to P10). Co-skewness values are estimated by using the rolling window of 36 monthy observations. Here, the decile portfolio P1 represent stocks with lowest (most negative) estimated ESK and the decile portfolio P10 contains stocks with lowest (most negative) estimated to reaching returns and all portfolios estimated to a monthly baservations. Here, the decile portfolio P1 represent stocks with lowest (most negative) estimated to present post ranking returns and all portfolios are rebalanced on a monthly baser. P1–P10 is the spread between the lowest co-skewness portfolio (P10). EW excess returns ($\%$ p.a) and VW excess returns ($\%$ p.a) represent amualized average monthly returns of equally weighted and value weighted portfolios respectively. MV (Rs. th) shows the average market value of stocks included in each portfolio and CAPM β is a full sample beta estimate of the value-weighted portfolio's returns. Letst value represents the results of the Wald test referring to the null hypothesis that there is no difference in means between the characteristics of P1 and P10.	stics of decile g to the CSK v . Here, the dec stimated CSK. 91–P10 is the s p.a) represent tocks included tocks included	portfolios values and a At month (spread betw annualized in each po e null hypo	based on (are assigned o P1 repres (+1), the ex (een the lov average m average m rtfolio and	co-skewnes d to ten diff sent stocks ccess return west co-ske onthly retu CAPM β i there is no	ss from Ja Terent por a with low ns of decil ewness po ewness po ewness po ewness po is a full sa difference	anuary 20 tfolios (P1 est (most 1 e portfolio ortfolio (P1 outfolio (P1 outfolio (P1 outfolio (P1 outfolio (P1) ample beta	00 to Dec to P10), (negative) s are estin) and hig phted and i estimate between t	cember 20 Co-skewn estimated nated to p hest co-sk value we of the val he charact	116. All st ess values CSK and resent post rewness pu ighted por ighted por tue-weight tue-weight	ocks liste are estimuthe decile tranking r ortfolio (P tfolios res red portfol	d in PSX ar ated by using p portfolio P eturns and a 10). EW ex ppectively. N io's returns.	te sorted in g the rolling 10 contains II portfolios cess returns AV (Rs. tn) t-test value
Table 2 Characteristics of decile portfolios based on co-kurtosis (CKT)	olios based	on co-ku	rtosis (C	KT)								

	Ρ1	P2	P1 P2 P3 P4	P4	P5	P6	$\mathbf{P7}$	P8	P5 P6 P7 P8 P9	P10	P10-P1	t-test
EW Excess Returns (% p.a)	0.224	0.202	0.218	0.255	5 0.271	0.132	0.115	0.318	0.393	0.669	0.445	0.961
VW Excess Returns (% p.a)	0.256	0.273	0.203	0.225	0.237	0.213	0.252	0.219	0.397	0.221	-0.035	-0.384
MV (Rs. tn)	2.959	3.289	3.509	3.484	4 3.131 3	3.201	3.435	3.896	4.635	5.425	2.466	4.746
CAPM β	0.164	0.000	-0.008	0.106	0.008	0.048	0.025	0.009	0.028	-0.060	-0.224	28.744
The above table presents characteristics of decile portfolios based on co-kurtosis from January 2000 to December 2016. All stocks listed in PSX are sorted in	ristics of de	scile portfo	lios based	on co-kurt	tosis from	January 2	000 to D	ecember 2	2016. All	stocks liste	d in PSX a	re sorted in

returns (% p.a) and VW excess returns (% p.a) represent annualized average monthly returns of equally weighted and value weighted portfolios respectively. MV (Rs. tn) shows the average market value of stocks included in each portfolio and CAPM β is a full sample beta estimate of the value-weighted portfolio's returns. t-test value represents the results of the Wald test referring to the null hypothesis that there is no difference in means between the characteristics of P1 and P10. ascending order at month (t) according to the CKT values and are assigned to ten different portfolios (P1 to P10). Co-kurtosis values are estimated by using the rolling window of 36 monthly observations. Here, the decile portfolio P1 represent stocks with lowest (most negative) estimated CKT and the decile portfolio P10 contains stocks with highest (most positive) estimated CKT. At month (t+1), the excess returns of decile portfolios are estimated to present post ranking returns and all portfolios are rebalanced on a monthly basis. P10–P1 is the spread between the highest co-kurtosis portfolio (P10) and lowest co-kurtosis portfolio (P10). EW excess

Table 1

	Ρl	P2	P3	P4	P5	P6	P5 P6 P7 P8	P8	\mathbf{P}	P10	P10 P10-P1	t-test
EW Excess Returns (% p.a)	-0.168	-0.168 -0.040	0.052	0.118	0.118 0.187 0.132 0.177 0.224 0.252	0.132	0.177	0.224	0.252	1.103	1.271	4.430
VW Excess Returns (% p.a)	-0.143	0.108	0.076	0.230	0.243	0.284 0.225	0.225	0.302	0.347	0.917	1.06	4.767
MV (Rs. tn)	0.112	5.091	6.466	5.860	4.545	4.098	4.77(3.140 2.	2.465	0.598	0.486	8.413
CAPM B	-0.032	-0.032 -0.119 -0.052 -0.014 -0.003 0.079 0.057 0.229 0.296 0.553	-0.052	-0.014	-0.003	0.079	0.057	0.229	0.296	0.553	0.585	24.021

	s y X II o e
	e sorted tfolio P1 ms and a (P1). EV spectively o's return 0.
	in PSX are timated by decile por unking retu a portfolio ortfolios res ortfolios res otro otro otro otro otro otro otro otr
	The above table presents characteristics of decile portfolios based on downside beta from January 2000 to December 2016. All stocks listed in PSX are sorted in ascending order at month (1) according to the DSB values and are assigned to ten different portfolios (P1 to P10). Downside beta values are estimated by using the rolling window of 36 monthly observations. Here, the decile portfolio P1 represent stocks with lowest (most negative) estimated DSB. At month (+1), the excess returns of decile portfolio ios are estimated DSB and the decile portfolio P10 portfolios are rebalanced on a monthly basis. P10–P1 is the sprease nombiny returns of decile portfolio ios are estimated portfolio (P1). EW excess returns (% p.a) represent annualized average monthly returns of equally weighted and value weighted portfolios respectively. W(Rs. m) shows the average market value of stocks included in each portfolio and CAPM β is a full sample beta estimate to prefloios returns. <i>AV</i> (Rs. m) shows the average market value of stocks included in each portfolio and CAPM β is a full sample beta estimate of the value-weighted portfolios returns. <i>Av</i> the resents the results of the wald test referring to the null hypothesis that there is no difference in means between the characteristics of P1 and P10.
	6. All stu- le beta vi mated D ted to pre lowest do d value v of the va the chara
	ember 201 p. Downsic gative) esti are estima P10) and reighted an a estimate s between
	0 to Deco 1 to P10 (most neg oortfolios oortfolio (equally w ample bet i mean
	uary 200 ttfolios (F n lowest of decile r ide beta r turns of o s a full so
	from Jan erent por ocks with returns c st downsi onthly re $(APM \beta i)$
	ide beta o ten diff present sl he excess the highe verage m verage m sis that th
	n downs ssigned t lio P1 rej h (t+1), t between ualized a ach portfo
	ss based and are a ile portfo . At mont e spread esent ann ded in e uded in e
	portfolic B values C the dec thed DSB -P1 is th p.a) repr ocks inclu eferring t
	of decile to the DSI ons. Here ve) estime pasis. P10 pasis. P10 alue of st
	ccteristics coording t observati ost positi nonthly t excess r market v s of the W
	mts chara mth (t) ac monthly ighest (m eed on a 1 ead VW and VW the result
	the preseder at mo by of 36 ks with hi rebalanc s (% p.a) shows the presents 1
J	above ta ending or- ing winds tains stoc folios are ess return (Rs. tn): st value re
	The asce roll con port MV <i>f</i> -tes

141

Asset Pricing Tests

Risk-adjusted performance

The descriptive results reported in the previous section clearly indicate that risk premium is associated with portfolios based on higher co-moments (CSK and CKT) and downside risk (DSB). In this section, Jensen's alpha, and Fama-French's alphas are estimated and reported to measure the risk-adjusted performance of equally weighted and value weighted portfolios (P1 to P10).

The following formula is used to estimate Jensen's alpha:

$$R_{i,t} - R_t^f = \alpha_i + \beta_{i,MKT} (R_{m,t} - R_t^f) + \varepsilon_{i,t}$$

where, $R_{i,t}$ is the return of portfolio (*i*) in a month (*t*), R_t^f is the risk-free rate at month (*t*), and $(R_{m,t}-R_t^f)$ is the excess return of market portfolio in a month (*t*). For Fama-French alpha, the Fama-French three-factor model (Fama & French, 1993) and Fama-French five-factor model (Fama & French, 2015) are used respectively:

$$R_{i,t} - R_t^f = \alpha_i + \beta_{i,MKT} (R_{m,t} - R_t^f) + \beta_{i,SMB} SMB_t + \beta_{i,HML} HML_t + \varepsilon_{i,t}$$

$$R_{i,t} - R_{t}^{f} = lpha_{i} + eta_{i,MKT}(R_{m,t} - R_{t}^{f}) + eta_{i,SMB}SMB_{t} + eta_{i,HML}HML_{t} + eta_{i,RMW}RMW_{t} + eta_{i,CMA}CMA_{t} + ela_{i,t}$$

Where, SMB_t is the size risk factor in a month (*t*), HML_t is the value risk factor in a month (*t*), RMW_t is the operating profitability risk factor in a month (*t*) and CMA_t is the investment risk factor in a month (*t*).

Moreover, to test the joint significance of decile portfolio alphas, the systems of equations have been used which are explained above. The importance of using a system of equation is that it helps to overcome the issue of measurement error in variables and the problems of heteroscedasticity and serial correlation can also be corrected if alphas are estimated by using generalized methods of moments (GMM).

Value Weighted Returns (VW Returns)

The alphas of value-weighted decile portfolios constructed on CSK, CKT and DSB are presented here. The result of Table 4 shows the alphas of CSK value weighted decile portfolios. It explains that even after the adjustment of commonly used risk factors, the CSK is not priced in PSX over and above market, size, value, profitability, and investment. The spread (P1–P10) yields statistically insignificant

abnormal performance of -8.84% p.a. (0.82), -11.79% p.a. (-0.86) and -19.07% p.a. (-0.86) under CAPM, Fama-French three-factor and Fama-French five-factor models respectively. Hence, we fail to prove that a high-risk premium is required by a prudent investor to invest in shares with negatively skewed returns in PSX. Furthermore, to gauge the understanding of asset pricing models of the time series behaviour of CSK portfolio and significance of overall model pricing errors, we have checked the joint significance of estimated alphas of decile portfolio through Wald test. The results again show that CSK decile portfolios unable to yield abnormal returns as we fail to reject the null hypothesis of joint estimates of alpha equal to zero because p-values (0.70, 0.89 and 0.46) of all three models is greater than 0.05. This shows that co-skewness is not considered as an additional risk source that is priced in PSX.

Table 5 explains the result of alphas of value-weighted decile portfolios constructed on the basis of CKT. The results reveal the significant evidence that co-kurtosis premium is not priced in PSX. The spread (P10–P1) yields an annualized Jensen alpha of 6.43% p.a. (0.61), 6.33% p.a. (0.49) and 13.96% p.a. (0.86) for all three asset pricing models. The result of the overall significance of estimated alphas through Wald test is also insignificant. Thus, it can be concluded that decile portfolios based on CKT also does not yield abnormal return and fails to prove that high-risk premium is required by a temperate investor to invest in shares with positive kurtosis returns in PSX. Hence, it can be concluded that CKT is not priced in PSX over and above market, size, value, profitability, and investment.

Table 6 explains the result of alphas of value-weighted decile portfolios constructed on the basis of DSB. Even after the adjustment of commonly known risk factors, the results documented in Table 3 remain intact and DSB is priced in PSX over and above market, size, value, profitability, and investment. The spread (P10-P1) for CAPM, Fama-French three-factor, and Fama-French fivefactor models yield annualized Jensen alpha of -79.47% p.a. (-3.37), -75.05% p.a. (2.54) and -87.45% p.a. (-2.40) respectively. The result of the overall significance of estimated alphas through Wald test is also significant as we reject the null hypothesis of joint estimates of alpha equal to zero. It can be said that decile portfolios based on DSB also yield abnormal return and prove that high-risk premium is required by a cautious investor to invest in shares in PSX. Hence, the alphas for all decile portfolios explain that CAPM fails to deliver efficient results in PSX which means there are factors that can better explain the portfolio returns other than a market factor. Thus, it can be concluded that financial analysts failed to capture variations in the financial market with the help of CAPM. Our results are in line with the critique given by Roll (1977) in which he argued that CAPM uses only individual asset returns as a market proxy for determining the riskreturn relationship whereas market portfolio is the combination of various factors.

	P1	P2	P3	P4	P5	P6	P7	P8	6d	P10	P10-P1	Wald-test
CAPM Alpha	-0.17	-0.87	9.32	6.73	3.48	12.56	0.98	7.99	-0.36	8.67	-8.84	7.24
	(0.02)	(0.14)	(1.14)	(0.88)	(0.60)	(1.12)	(0.18)	(1.44)	(0.00)	(1.20)	(0.82)	0.70
FF3 Alpha	-2.19	1.70	8.04	4.50	2.78	16.24	-0.64	7.68	-1.98	9.60	-11.79	4.98
	(0.24)	(0.21)	(0.80)	(0.47)	(0.39)	(1.21)	(60.0-)	(0.10)	(-0.27)	(1.06)	(-0.86)	0.89
FF5 Alpha	14.58	7.94	-2.02	-6.64	-6.13	3.16	-4.30	6.33	5.79	24.08	-19.07	9.77
	(1.17)	(0.89)	(-0.29)	(-1.14)	(-0.78)	(0.49)	(-0.62)	(0.93)	(0.61)	$(1.93)^{**}$	(-0.86)	0.46
The above table presents the risk-adjusted performance of the decile value weighted co-skewness portfolios. Data includes all shares listed on PSX from January 2000 till December 2016. Data is sorted in ascending order at month (<i>t</i>) to estimate co-skewness values by using 36 months rolling window of observations and all portfolios are rebalanced on a monthly basis. The decile portfolio P1 represent stocks with lowest (most negative) estimated co-skewness and the decile portfolio P1 contains stocks with	resents the ri bata is sorted onthly basis.	sk-adjusted 1 in ascendin The decile p	performance ig order at m ortfolio P1 r	of the decile ionth (t) to e epresent stoc	e value weig estimate co-s cks with lowe	hted co-skev kewness val est (most ne	wness portfol lues by using gative) estim	ios. Data in g 36 months ated co-skev	cludes all sh rolling win vness and th	nares listed on dow of observ e decile portfo	PSX from Javations and a olio P10 conta	Il portfolios are ains stocks with

Table 4

highest (most positive) estimated co-skewness and P1-P10 is the spread between lowest co-skewness portfolio (P1) and highest co-skewness portfolio (P10). CAPM alpha, F73 alpha, and FF5 alpha present the annualized estimation of alphas derived from CAPM, Fama-French three-factor, and Fama-French five-factor respectively. Results of e-statistics are reported in parentheses indicating the statistical significance at different levels (* at 10%, ** at 5% and *** at 1%). Finally, the Wald test reports chi-square statistics referring to the null hypothesis that all ten alphas of decile portfolios are jointly equal to zero and there p-values are in parenthesis below the statistic.

Ś
e
P
Ta

. ç . .

	Ρ1	P2	P3	P4	PS	P6	P7	P8	6d	P10	P10-P1	Wald-test
CAPM Alpha	11.65	10.82	2.04	2.19	1.16	0.13	6.49	1.56	20.71	5.21	6.43	9.18
	(1.63)*	(1.58)	(0.31)	(0.32)	(0.17)	(0.02)	(1.08)	(0.27)	(1.42)	(0.74)	(0.61)	0.52
FF3 Alpha	13.00	13.07	-0.87	0.70	2.27	-5.63	4.42	-3.62	22.01	6.67	6.33	8.12
	(1.48)	(1.55)	(-0.10)	(0.08)	(0.27)	(-0.81)	(0.59)	(-0.50)	(1.27)	(0.76)	(0.49)	0.62
FF5 Alpha	15.97	14.91	3.42	1.85	5.42	-3.36	4.98	-3.95	22.20	7.26	13.96	10.00
	(1.75)*	$(1.69)^{*}$	(0.39)	(0.21)	(0.60)	(-0.45)	(0.63)	(0.51)	(1.21)	(0.77)	(0.86)	0.49

highest (most positive) estimated co-kurtosis and P10–P1 is the spread between highest co-kurtosis portfolio (P10) and lowest co-kurtosis portfolio (P1). CAPM alpha, FF3 alpha, and FF5 alpha present the annualised estimation of alphas derived from CAPM, Fama-French three-factor, and Fama-French five-factor respectively. Results of The above table presents the risk-adjusted performance of the decire value weighted co-kurtosis portionos. Data includes an shares instea on F5A from January 2000 the December 2016. Data is sorted in ascending order at month (t) to estimate co-kurtosis values by using 36 months rolling window of observations and all portfolios are rebalanced on a monthly basis. The decile portfolio P1 represent stocks with lowest (most negative) estimated co-kurtosis and the decile portfolio P10 contains stocks with -statistics are reported in parentheses indicating the statistical significance at different levels (* at 10%, ** at 5% and *** at 1%). Finally, the Wald test reports chi-square statistics referring to the null hypothesis that all ten alphas of decile portfolios are jointly equal to zero and there p-values are in parenthesis below the statistic.

	P1	P2	P3	P4	P5	P6	P7	P8	6d	P10	P10-P1	Wald-test
CAPM Alpha	-15.65	3.30	-4.02	4.41	09.0	3.67	-1.07	5.03	10.34	63.81	-79.47	57.67
	$(10.19)^{***}$	(0.56)	(-0.98)	$(2.13)^{**}$	(1.12)	$(1.70)^{*}$	(-2.21)**	$(1.79)^{*}$	$(1.95)^{**}$	$(2.75)^{***}$	(-3.37)***	0.00
FF3 Alpha	-15.72	5.31	-3.15	2.25	-2.83	1.77	-3.44	3.17	6.32	59.33	-75.05	37.54
	$(-5.59)^{***}$	(0.73)	(-1.62)*	(1.36)	(-1.46)	$(2.80)^{***}$	(-0.56)	(1.40)	(1.55)	$(2.04)^{**}$	$(2.54)^{***}$	0.00
FF5 Alpha	-15.72	9.31	(-0.39)	3.09	(-2.01)	(0.95)	(-1.68)	3.12	7.82	62.24	-87.45	34.07
I	(-5.23)***	(1.25)	(0.07)	(1.47)	(1.31)	$(1.84)^{*}$	(-2.26)**	(1.38)	$(1.64)^{*}$	$(2.03)^{**}$	(-2.40)**	0.00

beta portfolios
bet
downside
ed
e-weight
ne
val
ofv
Alphas

Table 6

sorted in ascending order at month (*i*) to estimate downside beta values by using 36 months rolling window of observations and all portfolios are rebalanced on a monthly basis. The decile portfolio P1 represent stocks with highest (most positive) estimated downside beta and the decile portfolio P10 contains stocks with highest (most positive) estimated downside beta and P10-P1 is the spread between highest downside beta portfolio (P10) and lowest downside beta portfolio (P1). CAPM alpha, FF3 alpha, and FF5 alpha present the amualized estimation of alphas derived from CAPM, Fama-French three-factor, and Fama-French five-factor respectively. Results of t-statistics are reported in parentheses indicating the statistical significance at different levels (* at 10%), ** at 5% and *** at 1%). Finally, the Wald test reports chi-square statistics referring to the null hypothesis that all ten alphas of decile portfolios are jointly equal to zero and there p-values are in parenthesis below the statistic.

Cross-Sectional Analysis

The results of time series analysis clearly show that the Jensen and Fama-French alphas are the clear indication of the existence of higher returns of portfolios constructed on the basis of higher moments and downside beta which leads to cross-sectional analysis¹³ for the robustness of findings. In 2005, Cochrane gave an argument in favour of a cross-sectional analysis that cross-sectional regression tells whether the factor itself priced or not whereas time series analysis depicts that whether a certain factor help to price an asset or not. Therefore, it is analysed that how well the decile portfolios returns sorted on the basis of CSK, CKT, and DSB are explained by the most commonly used asset price model i.e. CAPM.

A cross-sectional regression of the excess portfolios on the portfolios beta is estimated as:

where, λ_0 represents the intercept, λ_1 is the estimated risk premium on the market, represents the measure of risk and ε_p is the standard error.

			F-test	<i>p</i> -value
Null Hypothesis: $= 0$				
Panel I: CSK	0.012	0.013	29.690	0.000
Panel II: CKT	0.013	0.006	25.320	0.000
Panel III: DSB	0.004	0.097	3.990	0.046
Null Hypothesis: =				
Panel I: CSK	0.012	0.013	0.000	0.958
Panel II: CKT	0.013	0.006	0.060	0.814
Panel III: DSB	0.004	0.097	12.98	0.000

Table 7 Cross-sectional test-CAPM

The result of Table 7 explained two hypotheses, the first hypothesis tests whether the intercept γ_0 is equal to zero i.e. $\gamma_0 = 0$ which if true shows that the risk factor beta explain the variation in portfolios return. Similarly, the second hypothesis tests whether γ_1 is equal to market excess return (R_m) which if accepted then it indicates the efficiency of the beta risk factor in explaining variation in the portfolio's returns. Hence, if both hypothesis are accepted then it means CAPM has been an efficient asset pricing model to capture risk and return relationship and it is verified.

The results of the first hypothesis clearly show that we significantly reject the null hypothesis that is $\gamma_0 = 0$ at 5% and 10% level of significance. On the other hand, the results of the second hypothesis show that we fail to reject the null hypothesis that is = for the beta, co-skewness, and co-kurtosis as their p-values are greater than 0.05 explaining that portfolios formed on these factors fail to capture market risk premium and the variation in market portfolio's return is efficiently capture by CAPM's beta only. On the other hand, the null hypothesis for downside beta is rejected as the p-value is less than 0.05 indicating that asset portfolios formed on the basis of downside beta efficiently capture market risk premium and beta is not the only factor that explains variation in market portfolio's return. Hence, reject CAPM for the entire period of analysis. Furthermore, a review of the literature suggested that to check the robustness of result least square approach only work under the normality assumption of assets returns. Hansen (1982) suggested that the Generalized Method of Moment (GMM) allows stationary ergodic distribution of asset returns at the cost of achieving asymtotic distribution. In other words, it allows for non-normality, conditional hetroscedasticity, and serial correlation. Moreover, Cochrane (2005) infers that the flexibility of incorporating misspecification in statistical models in distribution theory and evaluation of misspecification models is provided by the GMM framework. Therefore, in this study the GMM method is applied to check the robustness of cross-sectional and time series analysis by using the following regression equation:

$$\bar{r}_p = \lambda_0 + \lambda_1 \beta_p + \varepsilon_p$$

Constructing three moments restrictions for each asset where $\hat{\beta}_p = k$.

Table 8 shows the results of Hansen J-test and Wald test. These tests are applied to check whether the conditions are exactly identified (q = k) or overidentified (q > k). If the condition is (q = k) then it means that can be solved by its counterparts. So to test over identification restriction with the null hypothesis that (q > k) are valid we have applied the test of Hansen commonly known as J-test. The results of J-test show that its *p*-values are greater than 0.05 for CSK, CKT, and DSB-and hence we fail to reject the null hypothesis. Furthermore, the Wald test is also calculated from the asymptotic covariance matrix of the estimates of GMM. The results of Wald test indicate that its *p*-values are less than 0.05 for DSB, CSK and CKT which mean null hypothesis is rejected and we concluded that CAPM fails to explain risk and return relationship in asset pricing.

		Panel	Panel I: CSK				Panel II: CKJ	I: CKT				Panel I	Panel III: DSB		
I	LS Approach	oach	0	GMM Approach	ich	LS Approach	proach	G	GMM Approach	ich	LS Approach	proach	5	GMM Approach	ch
	Alpha	Beta	Alpha	Beta	Lambda	Alpha	Beta	Alpha	Beta	Lambda	Alpha	Beta	Alpha	Beta	Lambda
ā	0.009	-0.105	0.001	0.013	0.064	0.012	0.172	0.012	0.040	0.315	-0.019	-0.026	-0.018	-0.060	0.308
L	(-1.19)	(1.40)	(0.17)	(0.13)	(1.82)	(1.92)	(2.16)	(0.86)	(2.09)	(0.87)	(-10.19)	(-1.06)	(-10.40)	(-3.16)	(3.40)
ca.	0.009	0.065	0.039	0.056	0.064	0.015	0.007	0.015	-0.059	0.315	0.003	-0.113	-0.061	0.006	0.308
77	(0.73)	(1.37)	(0.92)	(0.61)	(1.82)	(2.46)	(0.0)	(2.43)	(-0.78)	(0.87)	(0.71)	(-1.86)	(-1.86)	(0.39)	(3.40)
50	0.016	-0.128	0.000	0.006	0.064	0.010	-0.001	0.010	-0.044	0.315	0.000	-0.045	0.002	-0.103	0.308
C I	(-1.36)	(2.25)	(0.14)	(0.10)	(1.82)	(1.50)	(-0.01)	(1.54)	(-0.57)	(0.87)	(90.0-)	(-0.85)	(0.38)	(-2.04)	(3.40)
ž	0.016	0.001	0.007	0.092	0.064	0.010	0.114	0.021	0.033	0.315	0.012	-0.007	0.015	-0.092	0.308
t	(0.01)	(2.17)	(1.05)	(1.16)	(1.82)	(1.27)	(1.45)	(0.72)	(0.81)	(0.87)	(2.14)	(60.0-)	(3.01)	(-1.06)	(3.40)
20	0.010	0.025	0.003	0.096	0.064	0.012	0.015	-0.010	0.037	0.315	0.013	0.004	0.017	-0.049	0.308
C	(0.31)	(1.70)	(0.68)	(1.25)	(1.82)	(1.67)	(0.15)	(0.39)	(0.84)	(0.87)	(1.98)	(0.04)	(2.63)	(-0.54)	(3.40)
2	0.017	0.153	0.012	0.229	0.064	0.010	0.055	0.019	0.032	0.315	0.015	0.086	-0.007	0.063	0.308
10	(1.18)	(1.74)	(1.32)	(2.21)	(1.82)	(0.66)	(1.52)	(0.70)	(0.85)	(0.87)	(0.95)	(2.21)	(-0.24)	(2.26)	(3.40)
70	0.008	0.110	0.033	0.080	0.064	0.013	0.032	0.013	0.016	0.315	0.011	0.064	-0.011	0.047	0.308
۲ /	(1.38)	(1.35)	(0.93)	(0.92)	(1.82)	(2.10)	(0.39)	(2.03)	(0.22)	(0.87)	(1.63)	(0.75)	(-0.35)	(1.96)	(3.40)
od	0.014	0.038	0.008	0.098	0.064	0.011	0.016	-0.002	0.032	0.315	0.015	0.237	0.020	0.151	0.308
0	(0.50)	(2.37)	(1.47)	(1.33)	(1.82)	(1.69)	(0.19)	(-0.08)	(0.87)	(0.87)	(2.46)	(1.98)	(2.81)	(1.52)	(3.40)
Q	0.007	0.020	0.002	0.059	0.064	0.025	0.036	0.027	0.091	0.315	0.017	0.305	0.069	0.082	0.308
6	(0.25)	(1.17)	(0.30)	(0.78)	(1.82)	(2.09)	(0.22)	(2.46)	(0.80)	(0.87)	(1.95)	(2.61)	(1.52)	(2.30)	(3.40)
010	0.012	0.081	-0.012	0.144	0.064	0.012	-0.053	0.011	-0.042	0.315	0.061	0.563	0.076	0.260	0.308
011	(0.93)	(1.86)	(-0.17)	(1.33)	(1.82)	(1.81)	(-0.62)	(1.57)	(-0.41)	(0.87)	(3.31)	(2.32)	(4.54)	(2.76)	(3.40)
Weld			7.880					13.410					149.330		
Valu			[0.6403]					[0.2017]					[0.000]		
					6.053					1.967					5.133
nalisell					[0.7346]					[0.992]					[0.8226]

and Day ose Co-Kurtosis Table 8 GMM Results_Co_Steam

Imran Umer Chhapra and Muhammad Kashif

CONCLUSION

Pricing a risk asset is one the most difficult part of an investor's life and empirical results of CAPM have to validate its failure in asset pricing which has motivated researchers to reexamine the basis on which this model has been developed. The empirical invalidity of linearity (symmetricity) of stock return and quadratic preference of investor in MV framework has given birth to Higher Moment CAPM and Downside Beta CAPM. Hence, many previous researchers have identified that co-skewness, co-kurtosis and downside beta are better predictors of asset returns than the traditional mean-variance framework.

To achieve the purpose of this study, decile portfolios are constructed on the basis of co-skewness, co-kurtosis and downside beta and *t*-test results reveal that only DSB is efficiently priced in PSX and the other two risk measure (CSK and CKT) fail to yield abnormal average returns and cannot be considered as additional risk source that is priced in PSX. Estrada (2002), Post and Vilet (2004), Ang, Xing and Chen (2006), Javid and Ahmad (2011), Foong and Goh (2012), Tahir et al. (2013) and Rashid and Hamid (2015) showed similar results in which they concluded that stocks that vary with the declining market are compensated with a high-risk premium for bearing downside risk. The results of time series and cross-sectional analysis also showed that CAPM does not significantly capture market risk premium and there is other risk measures also such as downside beta. Furthermore, the findings can help investors in ascertaining an appropriate risk measure in constructing well-diversified and efficient portfolios¹⁴. The results are also of great importance to firm managers as it will allow them to take appropriate decision related to capital budgeting process by efficiently estimating the cost of common equities in Pakistan. For future research implication, downside risk versions of co-skewness and co-kurtosis should be incorporated to determine whether downside higher order co-moments are priced in PSX or not and behavioural approach can be incorporated in this field to better understand the risk and return relationship in asset pricing.

NOTES

- 1. Also known as "Expected Returns-Variance of Returns" rule or Modern Portfolio Theory (MPT).
- 2. A portfolio which among all other portfolios, for the given mean return, has a minimum variance or for a given variance has a maximum mean return.
- 3. $R_t = Pt_1 Pt_0$, where *p* is price at t_0 and t_1 . At t_0 the investor decides the allocation of his resources and holds it till the period t_1 .

- 4. Indicating expected rate of return (mean) and expected risk of return (variance) of a security. Mean is desirable and should be maximise whereas variance is undesirable and should be minimised.
- 5. Used to measure firm size.
- 6. IARA shows the investors behaviour that is when the wealth of an investor increases he becomes more risk averse which portrays unrealistic human behaviour and contradicts daily life experiences.
- 7. CRRA utility explains that investment in risky asset has no relationship with investor's wealth. If the investor wealth will increase he will invest his whole wealth in riskless asset indicating no correlation of investor's income with risky asset returns. Hence, the demand for risky assets of a CARA investor will not be affected (Campbell & Viceira, 2002).
- 8. Semi-variance models are more complex than variance model and require twice the number of data inputs. Hence, before the invention of microcomputer in 1980s it was not possible to conduct complex analysis.
- 9. Developed by Bawa and Lindenberg (1977), Harlow and Rao (1989) and Estrada (2002).
- 10. In finance, the tendency to exclude failed companies or managers from performance evaluations or studies simply because they do not exist. Survivorship bias can result in skewed findings in a study and lead a casual reader to believe that a study shows a rosier picture than it really does.
- 11. Estrada (2002) downside beta considers the downside co-movement of asset returns and market returns in co-semi-variance whereas Bawa and Lindenberg (1977) and Harlow and Rao (1989) consider only the downside movement of market returns as risk. Hence, beta estimation may have an impact of such structural differences.
- 12. Percentiles portfolios are preferable when number of firms in dataset is less for portfolio construction resulting in more companies in a single portfolio. The negative of constructing percentile portfolios is lesser cross-sectional portfolio diversification.
- 13. Goyal (2012) explained that asset pricing models are inherently cross-sectional in nature as they naturally impose cross-sectional pricing restrictions and are more versatile than time series regression.
- 14. Carhart, Krail, Stevens and Welch (1996) explain efficient portfolios for asset pricing as portfolios that contain: (a) spread in expected returns, (b) spread in loading true factors to have diversification to risk factors, and (c) low cross-correlation and economically investable.

REFERENCES

- Alles, L. & Murray, L. (2017). Asset pricing and downside risk in the Australian share market. *Applied Economics*, 49(43), 4336–4350, https://doi.org/10.1080/000368 46.2017.1282143
- Ang, A., Xing, Y., & Chen, J. (2006). Downside risk. *Review of Financial Studies*, 19(4), 1191–1239. https://doi.org/10.1093/rfs/hhj035

- Ang, J. S., & Chua, J. (1979). Composite measures for the evaluation of investment performance. *Journal of Financial and Quantitative Analysis*, XIV(2), 361–384. https://doi.org/10.2307/2330509
- Arditti, F. D. (1967). Risk and the required return on equity. *The Journal of Finance*, 22(1), 19–36. https://doi.org/10.1111/j.1540-6261.1967.tb01651.x
- Athayde, G. M., & Flores, J. (2004). Finding a maximum skewness portfolio: A general solution to three-moments portfolio choice. *Journal of Economic Dynamics and Control*, 28(7), 1335–1352. https://doi.org/10.1016/S0165-1889(02)00084-2
- Ayub, U., Shah, S. Z. A., & Abbas, Q. (2015). Robust analysis for downside risk in portfolio management for a volatile stock market. *Economic Modelling*, 44, 86–96. https:// doi.org/10.1016/j.econmod.2014.10.001
- Bajpai, S., & Sharmab, A. K. (2015). An empirical testing of capital asset pricing model in India. *Procedia - Social and Behavioral Sciences*, 189, 259–265. https://doi. org/10.1016/j.sbspro.2015.03.221
- Balzer, L. (2001). Investment risk: A unified approach to upside and downside returns. In F. Sortino & S. Satchell (eds.). *Managing downside risk in financial markets* (pp. 103–155). Oxford: Butterworth-Heinemann. https://doi.org/10.1016/B978-075064863-9.50009-8
- Banz, R. W. (1981). The relationship between returns and market value of common stocks. *Journal of Financial Economics*, 9(1), 3–18. https://doi.org/10.1016/0304-405X(81)90018-0
- Barone-Adesi, G. (1985). Arbitrage equilibrium with skewed asset returns. *Journal* of Financial and Quantitative Analysis, 20(03), 299–313. https://doi.org/10.2307/2331032
- Bawa, V. S., & Lindenberg, E. B. (1977). Capital market equilibrium in a mean-lower partial moment. *Journal of Financial and Quantitative Analysis*, 12(4), 635. https://doi.org/10.1017/S0022109000023346
- Black, F. (1972). Capital market equilibrium with restricted borrowing. *The Journal of Business*, 45(3), 444–455. https://doi.org/10.1086/295472
- Bollerslev, T., Engle, R., & Wooldridge, J. (1988). A capital asset pricing model with timevarying covariances. *The Journal of Political Economy*, 96(1), 116–131. https:// doi.org/10.1086/261527
- Breeden, D. (1979). An intertemporal asset pricing model with stochastic consumption and investment opportunities. *Journal of Financial Economics*, 7(3), 265–296. https:// doi.org/10.1016/0304-405X(79)90016-3
- Campbell, J. Y., & Viceira, L. (2002). *Strategic asset allocation: Portfolio choice* for long-term investors. Oxford: Oxford University Press. https://doi. org/10.1093/0198296940.001.0001
- Carhart, M., Krail, R. J., Stevens, R. I., & Welch, K. E. (1996). *Testing the conditional CAPM*. Working paper, University of Southern California.
- Choudhry, T., & Wu, H. (2009). Forecasting the weekly time-varying beta of UK firms: GARCH models vs Kalman Filter method. *European Journal of Finance*, *15*(3–4), 437–444. https://doi.org/10.1080/13518470802604499

- Christie-David, R., & Chaudhry, M. (2001). Coskewness and cokurtosis in futures markets. *Journal of Empirical Finance*, 8(1), 55–81. https://doi.org/10.1016/ S0927-5398(01)00020-2
- Chunhachinda, P., Dandapani, K., Hamid, S., & Prakash, A. (1997). Portfolio selection and skewness: Evidence from international stock markets. *Journal of Banking and Finance*, 21(2), 143–167. https://doi.org/10.1016/S0378-4266(96)00032-5
- Collins, R. A., & Gbur, E. E. (1991). Borrowing behavior of the proprietary firm: Do some risk-averse expected utility maximizers plunge? Western Journal of Agricultural Economics, 16(2), 251–258.
- Copeland, T. E., Weston, J. F., & Shastri, K. (2005). *Financial theory and corporate policy* (4th Ed.). Boston: Addison-Wesley.
- Da, Z., Guo, R.-J., & Jagannathan, R. (2012). CAPM for estimating the cost of equity capital: Interpreting the empirical evidence. *Journal of Financial Economics*, 103(1), 204–220. https://doi.org/10.1016/j.jfineco.2011.08.011
- Dempsey, M. (2013). The Capital Asset Pricing Model (CAPM): The history of a failed revolutionary idea in finance? *Abacus: A Journal of Accounting, Finance, and Business Studies, 49*(Supplement), 7–23. https://doi.org/10.1111/j.1467-6281.2012.00379.x
- Dimson, E., Nagel, S., & Quigley, G. (2003). Capturing the value premium in the United Kingdom. *Financial Analysts Journal*, 59(6), 35–45. https://doi.org/10.2469/faj. v59.n6.2573
- Dittmar, R. F. (2002, February). Nonlinear pricing kernels, kurtosis preference, and evidence from the cross section of equity returns. *The Journal of Finance*, *57*(1), 369–403. https://doi.org/10.1111/1540-6261.00425
- Estrada, J. (2002). Systematic risk in emerging markets: The D-CAPM. *Emerging Markets Review*, 3(4), 365–379. https://doi.org/10.1016/S1566-0141(02)00042-0
- Estrada, J., & Serra, A. (2005). Risk and return in emerging markets: Family matters. *Journal of Multinational Financial Management*, 15(3), 257–272. https://doi. org/10.1016/j.mulfin.2004.09.002
- Fama, E. F. (1965). The behavior of stock-market prices. *The Journal of Business*, 38(1), 34–105. https://doi.org/10.1086/294743
- Fama, E. F., & French, K. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics*, 33(1), 3–56. https://doi.org/10.1016/0304-405X(93)90023-5
- Fama, E., & French, K. (2004). The capital asset pricing model: Theory and evidence. Journal of Economic Perspective, 18(3), 25–46. https://doi.org/10.1016/0304-405X(93)90023-5
- Fama, E., & French, K. (2015). A five-factor asset pricing model. Journal of Financial Economics, 116(1), 1–22.
- Fang, H., & Lai, T. (1997). Co-kurtosis and capital assets pricing. *The Finance Review*, 32(2), 293–307. https://doi.org/10.1111/j.1540-6288.1997.tb00426.x
- Foong, S. S., & Goh, K. L. (2012). Determinants of sectoral cost of equity for an emerging market: The case of Malaysian firms. *Capital Market Review*, 20(1&2), 21–41.
- Galagedera, D., & Jaapar, A. (2009). Modeling time-varying downside risk. *The IUP Journal of Financial Economics*, 7(1), 36–51.

- Galagedera, D., Maharaj, E., & Brooks, R. (2008). The relationship between downside risk and return: New evidence through a multiscaling approach. *Applied Financial Economics*, 18(20), 1623–1633. https://doi.org/10.1080/09603100701720435
- Gollier, C., & Pratt, J. W. (1996). Risk vulnerability and the tempering effect of background risk. *Econometrica*, 64(5), 1109–1123. https://doi.org/10.2307/2171958
- Goyal, A. (2012). Empirical cross-sectional asset pricing: A survey. Financial Markets and Portfolio Management, 26, 3–38. https://doi.org/10.1007/s11408-011-0177-7
- Grauer, R. R. (2002). Asset pricing theory and tests. Cheltenham, U.K.: Edward Elgar Publishing Inc.
- Grootveld, H., & Hallerbach, W. (1999). Variance vs downside risk: Is there really that much difference? *European Journal of Operational Research*, *114*(2), 304–319. https://doi.org/10.1016/S0377-2217(98)00258-6
- Hansen, L. (1982). Large sample properties generalized method of moments estimators. *Econometrica*, 50(4), 1029–1054. https://doi.org/10.2307/1912775
- Harlow, W. V., & Rao, R. K. S. (1989). Asset pricing in a generalized mean-lower partial moment framework: Theory and evidence. *The Journal of Financial and Quantitative Analysis*, 24(3), 285–311. https://doi.org/10.2307/2330813
- Harvey, C. R., & Siddique, A. (2000). Conditional skewness in asset pricing tests. *The Journal of Finance*, 55(3), 1263–1295. https://doi.org/10.1111/0022-1082.00247
- Hwang, S., & Satchell, S. E. (1999). Modeling emerging market risk premia using higher moments. *International Journal of Finance & Economics*, 4(4), 271– 296. https://doi.org/10.1002/(SICI)1099-1158(199910)4:4%3C271::AID-IJFE 110%3E3.0.CO;2-M
- Javid, A. Y., & Ahmad, E. (2008). Test of multi-moment capital asset pricing model: Evidence from Karachi Stock Exchange. Pakistan Institute of Development Economics (PIDE) Working Papers, 49, 1–28.
- Javid, A. Y., & Ahmad, E. (2011). Asset pricing behaviour with dual-beta in case of Pakistani stock market. *The Pakistan Development Review*, 50(2), 95–118. https:// doi.org/10.30541/v50i2pp.95-118
- Jondeau, E., & Rockinger, M. (2006). Optimal portfolio allocation under higher moments. *European Financial Management*, 12(1), 29–55. https://doi.org/10.1111/j.1354-7798.2006.00309.x
- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, 47(2), 263–292. https://doi.org/10.2307/1914185
- Kendall, M. G., & Hill, A. B. (1953). The analysis of economic time-series Part 1: Prices. *Journal of the Royal Statistical Society, Series A*, 16(1), 11–34. https://doi. org/10.2307/2980947
- Kimball, M. (1990). Precautionary saving in the small and the large. *Econometrica*, 58(1), 53–73. https://doi.org/10.2307/2938334
- Kimball, M. (1992). Precautionary motives for holding assets. In P. Newman, M. Miligate,
 & J. Falwell (eds.). *The new palgrave dictionary of money and finance* (pp. 158–161). London: MacMillan. https://doi.org/10.3386/w3586
- Klemkosky, R. C. (1973). The bias in composite performance measures. *Journal of Financial* and Quantitative Analysis, 8(3), 505–514. https://doi.org/10.2307/2329649

- Kon, S. (1984). Models of stock returns: A comparison. *The Journal of Finance*, *39*(1), 147–165. https://doi.org/10.1111/j.1540-6261.1984.tb03865.x
- Kostakis, A., Muhammad, K., & Siganos, A. (2012). Higher co-moments and asset pricing on the London Stock Exchange. *Journal of Banking & Finance*, 36(3), 913–922. https://doi.org/10.1016/j.jbankfin.2011.10.002
- Kraus, A., & Litzenberger, R. H. (1976). Skewness preference and the valuation of risk assets. *The Journal of Finance*, 31(4), 1085–1100. https://doi. org/10.1111/j.1540-6261.1976.tb01961.x
- Libby, R., & Fishburn, P. (1977). Behavioral models of risk taking in business decision: A survey and evaluation. *Journal of Accounting Research*, 15(2), 272–292. https:// doi.org/10.2307/2490353
- Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budget. *The Review of Economics and Statistics*, 47(1), 13–37. https://doi.org/10.2307/1924119
- Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36(4), 394–419. https://doi.org/10.1086/294632
- Mao, J. C. (1970). Models of capital budgeting, E-V vs. E-S. Journal of Financial and Quantitative Analysis, 5(5), 657–676. https://doi.org/10.2307/2330119
- Markowitz, H. (1952). Portfolio selection. *Journal of Finance*, 7(1), 77–91. https://doi. org/10.1111/j.1540-6261.1952.tb01525.x
- Markowitz, H. M. (1959). Portfolio selection: Efficient diversification of investments. *The Journal of Business*, 33(4), 391–393. https://doi.org/10.1086/294377
- Mergner, S., & Bulla, J. (2008). Time-varying beta risk of Pan-European industry portfolios: A comparison of alternative modeling techniques. *The European Journal of Finance*, 14(8), 771–802. https://doi.org/10.1080/13518470802173396
- Merton, R. C. (1973). An intertemporal capital asset pricing model. *Econometrica*, 41(5), 867–887. https://doi.org/10.2307/1913811
- Nagel, S. (2001). Accounting information free of selection bias: A new UK database 1953– 1999. Working paper, Stanford Graduate School of Business. Avaiable at SSRN. Retrieved 1 May 2013, from http://ssrn.com/abstract=28672
- Post, T., & Vilet, P. V. (2004). Conditional downside risk and the CAPM. Working Report, Erasmus Research Institute of Management (ERIM), Rotterdam School of Management/ Rotterdam School of Economics, Erasmus Universities Rotterdam, 1–31.
- Quirk, J., & Saposnik, R. (1962). Admissibility and measurable utility functions. *Review of Economic Studies*, 29(2), 140–146. https://doi.org/10.2307/2295819
- Ranaldo, A., & Favre, L. (2005). Hedge fund performance & higher-moment market model. *Journal of Alternative Investment*, 8(3), 37–51. https://doi.org/10.3905/ jai.2005.608031
- Rashid, A., & Hamid, F. (2015). Downside risk analysis of returns on the Karachi Stock Exchange. *Managerial Finance*, 41(9), 940–957. https://doi.org/10.1108/MF-09-2014-0245
- Rashid, A., & Mehmood, A. (2018). Downside risk analysis of returns of financial institutions in Pakistan. *Indonesian Journal of Management and Business Economics*, 1(1), 1–18. https://doi.org/10.32455/ijmbe.v1i1.20

- Raza, H. (2018). Is D-CAPM superior to CAPM? The case of Pakistan Stock Exchange. NUML International Journal of Business & Management, 13(1), 96–106.
- Roll, R. (1977). A critique of the asset pricing theory's tests Part I: On past and potential testability of the theory. *Journal of Financial Economics*, 4(2), 129–176. https:// doi.org/10.1016/0304-405X(77)90009-5
- Roy, A. D. (1952). Safety-first and the holdings of assets. *Econometrica*, 20(3), 431–449. https://doi.org/10.2307/1907413
- Schneider, P., Wagner, C., & Zechner, J. (2016). Low-risk anomalies? CFS Working Paper, no. 550, Center for Financial Studies, Goethe-Universität House of Finance Frankfurt. https://doi.org/10.2139/ssrn.2858933
- Scott, R. C., & Horvath, P. (1980, Sept). On the direction of preference for moments of higher order than the variance. *The Journal of Finance*, 35(4), 915–919. https:// doi.org/10.1111/j.1540-6261.1980.tb03509.x
- Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under condition of risk. *The Journal of Finance*, 19(3), 425–442. https://doi. org/10.1111/j.1540-6261.1964.tb02865.x
- Shumway, T. (1997). The delisting bias in CRSP data. *The Journal of Finance*, 52(1), 327–340. https://doi.org/10.1111/j.1540-6261.1997.tb03818.x
- Soares, N. & Stark, A. W. (2009). The accruals anomaly Can implementable portfolio strategies be developed that are profitable net of transactions costs in the UK? *Accounting and Business Research*, 39(4), 321–345. https://doi.org/10.1080/000 14788.2009.9663371
- Tahir, M., Abbas, Q., Sargana, S. M., Ayub, U., & Saeed, S. K. (2013). An investigation of beta and downside beta based CAPM: Case study of Karachi Stock Exchange. *American Journal of Scientific Research*, 85, 118–135. https://doi.org/10.2139/ ssrn.2241416
- Tufano, P. (2008). Saving whilst gambling: An empirical analysis of U.K. premium bonds. American Economic Review, 98(2), 321–326. https://doi.org/10.1257/ aer.98.2.321
- Vasant, J., Irgolic, L., Rajaratnam, K., & Kruger, R. (2014). A comparison of meanvariance and mean-semivariance optimisation on the JSE. *The Journal of Applied Business Research*, 30(6), 1587–1596. https://doi.org/10.19030/jabr.v30i6.8876